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CONTENTS

Physics. — *The Mechanism of Mountain-formation in Geosynclinal Belts.* By F. A. VENING MEINESZ.

(Communicated at the meeting of April 29, 1933).

The gravity survey of the Netherlands East Indies in 1929 and 1930 has shown the existence of a narrow belt of strong negative gravity anomalies accompanied on both sides by fields of positive anomalies. On several occasions the writer has already mentioned that the course of this belt prevents looking for the cause of these anomalies either in uncompensated surface features or in an uncompensated recent sinking of the Earth's surface in this belt. We cannot find another explanation than the assumption of a great mass-defect in the upper layers of the Earth and this defect is so large that it is impossible to localize it in the superficial layers of the crust; we should have to assume densities so small that they are practically out of the question.

This has brought the writer to assume a thickening of the crust by a downward root, which takes the place of the denser sima layer below the crust and in this way causes the lack of gravity attraction. Assuming a difference of the densities of the crust and of the sima of 0.5, we find dimensions of this root of more than 1000 Km^2 vertical cross-section in profiles perpendicular to the axis of the belt. If there should be a second density-discontinuity in the crust itself, part of the mass-defect will occur in this latter boundary by means of a similar protuberance of the upper in the denser lower layer.

As the Netherlands East Indies belong to the Alpine-Himalayan geosynclinal belt and as the frequency of earthquakes indicates that the orogenic phenomena are still continuing, it is obvious to connect the formation of the root with those phenomena. This is confirmed by the fact that the great majority of the earthquake-centra are located in or near the belt of negative anomalies. This assumption has led the writer to the supposition that the crust under the action of great horizontal stresses is buckling inwards along the axis of this belt and that the folding and overthrusting of the surface layers, as found by the geologists, are an accompanying feature of this great phenomenon. The thickness of the crust is assumed to be of the order of 25 Km.

The further elaboration of this assumption and the consideration of the geological facts has led to the following tentative hypothesis about the history of geosynclinal belts. By some cause that is yet unknown, the Earth's crust in these belts appears to be subject to varying horizontal stresses of great magnitude. These stresses begin by bringing about an

elastical compression of the crust, eventually accompanied by metamorphism of the rocks in denser modifications, and the resulting density increase gives a sinking of the belt because of the tendency towards reestablishment of isostatic equilibrium. The load on the surface, brought about by the subsequent sedimentation, will further promote this sinking tendency.

Gradually the moment will come that the stresses attain a certain limit and, when further increasing, the crust will give way by buckling inside. The objection that it will not do this inwards, where the resistance is so much greater than outwards, may be met by the consideration of the kind of resistance offered by the simatic layer; this resistance is mainly or wholly viscous and so it becomes negligible incase the movement is slow enough. On the other hand the energy to be spent because of the disturbance of gravimetric equilibrium is much smaller when buckling inwards than outwards and the initial deformation is also promoting this direction of the buckling movement.

While the main crust is pressed inwards and is forming a downward protuberance, the superficial layer is squeezed together and is protruding upwards, forming eventually island rows in the synclinal basins. In the basins sedimentation will continue.

In the downward protuberances, the crust is getting in regions where the temperature is higher and so we have to expect that its plasticity will increase and that it will melt on the long run. This has two consequences, an increase in volume, which must bring about a rising of the crust, and a gradual flowing away along the lower boundary of the crust. The supposition of a crustal rising in a period following on the folding period is in harmony with the geological facts; most mountain chains show epirogenetic rising in that period and a great many geomorphologists are even inclined to ascribe the greatest part of the topographic elevation of the chain to this subsequent uprising.

The lateral flowing away of at least part of the downward protuberance, widens the root and this agrees with the gravity results found in older mountain-chains; the Alps, for instance, show a broader root than the belt in the Indies indicates, although even in the Alps the anomalies indicate a root that is slightly narrower than the mountain-system. This widening of the root must bring along widening of the crustal rising.

The last stage of the whole phenomenon must be the solidification of the root and this will have the contrary effect to the melting: It brings about contraction and sinking of the mountain-system. It is, therefore, not only the erosion, which causes the disappearance of the mountains, but part may be due to this sinking effect. This is likewise in harmony with the facts. JEFFREYS has pointed out the difficulty in assuming the erosion as the only cause of the lowering of the mountain-ranges, because in this case the erosion should not only have to take away the visible mountain-masses but also the deeper masses, that would rise because of the reestablishment

of isostatic equilibrium; the result would be to bring deep layers to the surface and this is contrary to the experience in old peneplained areas where formerly ranges have existed. JEFFREYS tries to explain this fact by assuming that a lower crustal layer, his "intermediary layer", is squeezed away under the weight of the mountains.

The above hypothesis can likewise make it clear. The widening of the root can already give an explanation, because the spreading of the root over a greater area implies its getting less deep and so the rising, brought about by reestablishment of isostasy, is reduced. The solidification of the root gives a second cause, because the sinking of the crust still further reduces the amount that has to be eroded for peneplaining the range. Together these effects can explain that no deep layers of the crust are coming to the surface during the peneplanation.

A thorough study of all geological and morphological facts of geosynclinal belts will be desirable for testing our hypothesis. As far as the Netherlands East Indies are concerned, the writer feels indebted to UMBGROVE for taking up this subject. In an elaborate study, of which the results are published in the "Tijdschrift v/h Koninklijk Nederlandsch Aardrijkskundig Genootschap", part 6, 1932, he has critically examined the geological evidence of the Archipelago since the beginning of the tertiary. The comparizon of his results with the gravity results gives valuable indications about a number of subjects. They will be treated at large in another publication. Here I wish only to mention the evidence that is directly connected with the narrow belt of strong negative gravity anomalies, which has been interpreted as the axis of the inward buckling of the crust. It sheds important light on our problem.

It appears that those islands that are situated in the belt, are the only islands that show great crustal folding and overthrusting since the beginning of the tertiary; this has been stated in Timor, in Ceram, in the Key Islands, in Buru and in E. Celebes, the other islands of the belt are not yet sufficiently investigated to know. The strike of the folding coincides with the axis of the belt. These facts seem to give a convincing confirmation of the hypothesis.

There is, however, a remarkable fact concerning these foldings and overthrustings, which merits our attention and discussion. It regards the data of these phenomena. They are all rather old, i.e. they occurred in the beginning of the miocene, save the folding in E. Celebes which is slightly later, in the middle of the miocene; this last area is an embranchment of the belt, which has obviously come into being after the origin of the main belt. Now I don't think it will do to assume that this belt has been inactive since this period although the geological evidence of the above islands does not show much; the first period after the folding shows rising and denudation, the pliocene brings sinking and sedimentation combined with the formation of "graben" in the central parts and the pleistocene rising again. But the earthquake-centres along the whole belt and the violence of

these quakes show that it is still the seat of active tectonic deformation; I don't think that this activity could be explained by secondary phenomena as for instance the reestablishment of isostasy by vertical movements. The only remaining possibility, which I should be inclined tentatively to assume as true, is a continuation of the crustal buckling without the upper layers of the crust in these islands being much affected. We have then to assume that the crust is pushed under the islands and disappears downwards. This seems a rather hypothetical assumption but I think we may find confirmation from another side, i.e. by a comparizon with the formation of the Alps, and if this is true it sheds important light on the problem of mountain-formation.

If we study the profiles through the Alps of RUD. STAUB and the publications of HEIM, LUGEON, ARGAND and others concerning the mechanism of the great folding and overthrusting phenomena, we see that there are two kinds of folds. In the Alps we have the great folds of the Austrides and the Pennides in the central zone, the Helvetides on the Northern side and the Dinarides on the southern side of this central zone. I should wish to consider here especially the West Alps with the Pennides in the central zone and the Helvetides to the North of them.

The character of these two groups of folds is clearly different. The Pennides, i.e. the St. Bernhard fold, the Monte Rosa fold and the Dent Blanche fold are much older than the Helvetides; they date from the cretaceous or perhaps even from the jura (JENNY a.o.) while the Helvetides date from the middle tertiary. The Pennides have the character of folds. i.e. both limbs of the fold are often well-developed and the deformation has a plastic type. Although their horizontal dimensions are greatest, their vertical dimensions are likewise considerable. The original sedimental layers, that have formed these folds must have been thick and homogeneous, in many cases the thickness must have been more than 5000 meters. Obviously we have to do here with the first folds formed in the geosyncline after the main crust had given way and the horizontal squeezing of the crust had started. I don't think it can be denied that the shape which the folds assumed can be better explained by the hypothesis that the sedimental layer was carried along with the main crust, which moved from both sides towards a buckling zone where it pushes downward, than in the old way by assuming that the sedimental layer was more or less independently pushed together and folded in an originally horizontal fold, that was squeezed upwards and downwards in a later stage of the phenomenon. I don't wish, however, to go here in details about the formation of these folds nor do I wish to offer any explanation about the mechanism that caused them; I only wish to suggest that their formation may probably be compared to the formation in the early miocene of the folds and overthrusts in Timor and in the other islands in the East Indies over the orogenic belt.

The Helvetides have another character. They are less like folds and more

like overthrusts. There is no middle limb connecting the crown of the upper limb with the inner edge of the lower one and it is even improbable that these two edges have been far apart in their original position as parts of the unbroken sedimental layer before the deformation took place, because this layer must have had a gradual change in thickness and facies from a neritic one in the North to a bathyal one in the South, and the inner and outer edges of consecutive sheets don't show any marked differences in thickness or facies.

The shape of the Helvetide nappes is much less deviating from the horizontal than that of the Pennides and the deformation has more the character of faulting than folding. Another difference worth mentioning is that they are decidedly more variable in facies than the Pennides.

It is generally accepted that the Helvetides are the sedimental cover of the European shelf against which the Alpine folding found its resistance, and that this cover was stripped off and pushed northwards by the action of the Pennide folds which partly have overridden them. It is here that I think that another view is possible and promising when we compare the phenomenon with that of the East Indies.

When we consider our conclusion about what is passing now in the orogenic belt of the East Indies, where we found it probable that the earth's crust is pushing laterally under the islands of this belt, and when we bring this conclusion in connection with the evidence of the Helvetides in the Alpine region, it seems logical to try whether we have not had the same thing there. We can consider the Helvetides also as underthrusts under the Pennide masses instead of the Pennides being thrust over the Helvetide nappes and the mechanical base of this view appears to me to be even more satisfactory. We come then to the following sequence of events.

In the first stages of the phenomenon the Pennide folds are formed by the squeezing together of the thick sedimentary layer of the geosyncline. The next period is a period of quiescence and the range formed by these folds rises upwards to great elevation above sea-level. Erosion causes sedimentation in the adjacent seas which are deep next to the range and less deep at a greater distance. In a later period the horizontal compression of the crust is increasing again and renewed movements take place. The main crust is moving towards the range and pushing under it towards the buckling zone where it disappears downwards and the sedimental layer is carried along and is likewise pushed under the range. In this way the first Helvetide underthrust is formed.

In the following periods this phenomenon is repeated a number of times. With each period of quiescence the range rises and with each subsequent period of compression a new sedimental sheet is pushed under the former formations and a new underthrust has come into being. It is clear that this way of explaining these thrusts agrees with the facts that have been given for the Helvetides: their character of thrusts instead of folds, the fact that middle limbs are never occurring, the variation from bathyal facies to

neritic facies from South to North and lastly their variable facies. As to this last point we can easily understand that the sedimentation in the Archipelago in the period that the sediments of these layers have been formed, was more varied than in the first period of the geosyncline, when the whole area was submerged. The intricate character of the Indian Archipelago may probably well be compared with the situation of that period in the Alpine region.

Besides the attraction of parallellizing the actual stage of the East Indies with a special stage of the Alpine history, this hypothesis about the way of formation of the Helvetides has, I think, another advantage. It appears to me that it can be better understood that the Helvetide nappe is carried along with the main crust and is pushed, together with this crust, under the existing mountain mass, than to assume in the way as it is done nowadays, that this relatively thin sheet is pushed over the crust by the mountain-mass behind it. I could not well imagine that in this case this sheet should not completely crumple up. Although it is true that it has done this in many places, as for instance in the Säntis-group, I should think that it ought to have done this in a much greater proportion and I should doubt even whether it could have been pushed at all over these great distances from the origin of the pushing power.

A question arises. What can be the cause of the rising tendency of the belt that appears to exist in the quiescent periods of the orogenic phenomenon? Could this be brought about by the rise of temperature and a beginning of melting of the root, which the previous buckling stage has brought into being? Or is it caused by the expansion of the crust because of a decrease of the compressional stress, and a resulting tendency towards reestablishment of isostasy? The writer feels inclined to think that both causes are cooperating in bringing about this effect.

Further study will be necessary before the above hypothesis can be considered to be well-founded. Further study will likewise be needed for adapting it to the East-Alps and to other mountain-systems. In all cases I think that more attention will have to be given to the part played by the main crust in these phenomena than has been done up to the present moment.

The gravity results found at sea and on land point, I think, to the fact that the inward buckling of the main crust, as has been assumed in the East Indies, has been a rather common occurrence in the history of the Earth's crust. In another paper, I hope to come back to this question and to give the facts that seem to point in this direction.

Physics. — *The melting-curve of neon to 200 kg/cm²*. By W. H. KEESOM and J. H. C. LISMAN. (Communication N⁰. 224*b* from the KAMERLINGH ONNES Laboratory at Leiden).

(Communicated at the meeting of April 29, 1933).

Summary. The melting-curve of neon was determined in the range from the triple-point to 198.4 kg/cm² and 27.42° K. We used a cryostat in which the hydrogen boils under a pressure of some atm.

§ 1. *Introduction.* In 1929 SIMON, RUHEMANN, and EDWARDS [1]) determined the melting-curve of neon up to 4900 kg/cm² and 69.1° K. Our apparatus, with which we measured the melting-curve of hydrogen [2]), enabling us to make more accurate measurements, we decided to carry out a new investigation of the melting-curve of neon. Our measurements cover the range from the triple-point to 198.4 kg/cm² and 27.42° K.

§ 2. *Method and apparatus.* The method followed [3]) and the cryostat used [4]) were described in previous papers. Temperatures were measured with the platinum thermometer *Pt* 24′ and pressures were read on a metal manometer which had been calibrated with a pressure balance.

The neon was kindly supplied by the PHILIPS' Lampworks, Eindhoven [5]) and had been purified thoroughly by freezing with solid hydrogen [6]).

The apparatus was filled as follows: some 15 L of very pure neon was frozen, by means of liquid hydrogen, in a small copper tube, connected with the apparatus. After closing the cock through which the neon had been supplied, the tube was brought at room temperature, while the neon evaporated into the apparatus, giving an initial pressure of some 25 kg/cm².

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1) F. SIMON, M. RUHEMANN and W. A. M. EDWARDS, Zs. f. phys. Chem. B **6**, 331, 1930.

2) W. H. KEESOM and J. H. C. LISMAN, These Proceedings **35**, 607, 1932. Comm. Leiden N⁰. 221*a*.

3) H. KAMERLINGH ONNES and W. VAN GULIK, These Proceedings **29**, 1184, 1926. Comm. Leiden N⁰. 184*a*.

4) W. H. KEESOM and J. H. C. LISMAN, These Proceedings **34**, 602, 1931. Comm. Leiden N⁰. 213*f*.

5) We are greatly indebted to the PHILIPS' Lampworks and to chem. docts. H. FILIPPO, Chief Engineer of this firm, for their kind assistance.

6) We render our cordial thanks to Messrs. H. VAN DIJK and J. A. VAN LAMMEREN, assistants at the KAMERLINGH ONNES Laboratory, for purifying the neon.

For the measurements of the two highest points of the melting-curve, a small quantity of neon had to be added. As for neon the heat of fusion per cm^3 is much greater than e.g. for hydrogen [1]), melting and solidification went on much more slowly. So for getting an accurate determination of each melting point, several measurements were necessary now.

§ 3. The *results* of the measurements are given in table I. We include the triple-point previously measured in this laboratory. The pressures are given in kg/cm^2 reduced to normal gravity. Fig. 1 shows the melting-curve.

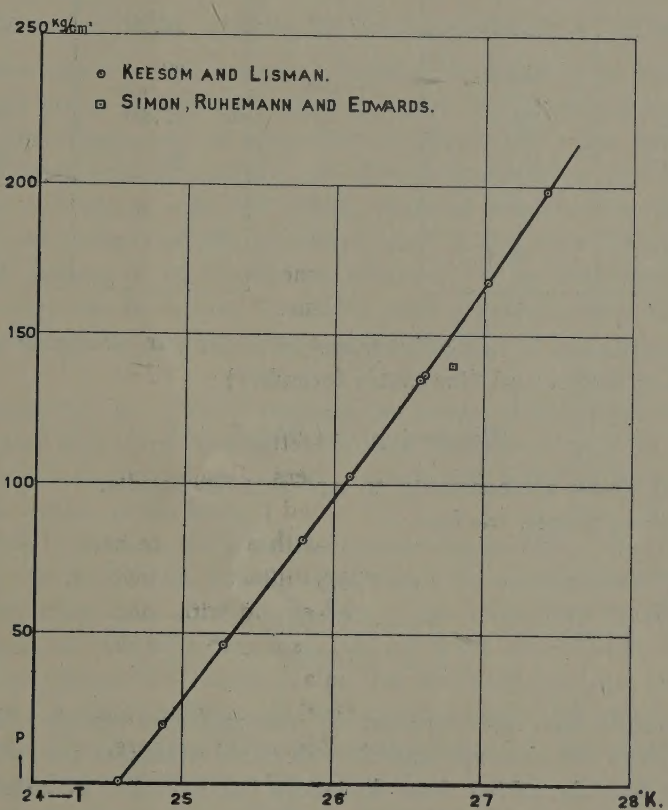

Fig. 1.

The agreement with the measurement of the triple-point by CROMMELIN and GIBSON is good. So is also the agreement with CLUSIUS' value of the normal melting point: 24.59° K. [2])

SIMON, RUHEMANN and EDWARDS [3]) found a melting-pressure of 140

[1]) Cf. W. H. KEESOM and H. KAMERLINGH ONNES, These Proceedings **20**, 1000, 1918. Comm. Leiden N⁰. 153*a*.

[2]) K. CLUSIUS, Zs. f. phys. Chem. B **4**, 1, 1929.

[3]) F. SIMON, M. RUHEMANN and W. A. M. EDWARDS, l. c.

kg/cm² at 26.8° K. This point does not agree very well with our curve, but it should be considered that the relative accuracy of SIMON and his cowor-

TABLE I.

T °K.	p kg/cm².
24.57 [1]	0.43
24.87	19.5
25.27	46.4
25.79	81.7
26.11	102.7
26.57	135.2
26.60^5	136.5
27.01^5	168.0
27.42	198.4

TABLE II.

T °K.	p kg/cm².
25	28.1
25.5	61.9
26	95.4
26.5	130.2
27	166.9
27.5	204.5
28	242.7

kers' measurements is smaller at these pressures than at higher ones. We have chosen SIMON and GLATZEL's formula [2]) :

$$^{10}\log (a+p) = c\ ^{10}\log T + b,$$

where a, b, and c are constants, to represent our results.

Using three points, we find:

$$a = 728.8$$
$$b = -0.16875$$
$$c = 2.18038.$$

The formula does not represent the measurements exactly; the differences between the observed and the calculated pressures are systematical. With this formula, taking these deviations into account, we calculated the melting-pressures of Table II.

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1) Triple-point: C. A. CROMMELIN and R. O. GIBSON, These Proceedings **30**, 362, 1927. Comm. Leiden N⁰. 185*b*.

2) F. SIMON and G. GLATZEL, Zs. f. anorg. u. allgem. Chem. **178**, 309, 1929.

Physics. — *On the supraconductivity of aluminium.* By W. H. KEESOM. (Communication N⁰. 224c from the KAMERLINGH ONNES Laboratory at Leiden).

(Communicated at the meeting of April 29, 1933).

Summary. It was found that aluminium becomes supraconductive at 1,14° K.

§ 1. *Introduction.* In preparing some experiments to be made in the temperature range of about 1.1° K to about 0.75° K, corresponding with helium vapour pressures of about 0.4 to about 0.007 mm mercury, an aluminium wire from HARTMANN and BRAUN appeared to become supraconductive already at a helium vapour pressure of somewhat more than 0.3 mm. Considering that the transition point of this wire lies rather near to that of gallium (0.24 mm helium pressure [1])), an aluminium wire of greater purity was investigated together with a gallium wire. The results are communicated in the following paragraph.

§ 2. *Results.* *a.* Table I contains the results of the measurements on an aluminium wire, furnished by HARTMANN and BRAUN in 1930, diameter 0.1 mm, resistance at 20° C : 20 Ω. Current and potential wires were of aluminium welded to the wire.

b. Table II contains the results of measurements on:

α. the wire Al-1924-II. It had been made of the same wire as Al-1924-I, drawn from aluminium kindly furnished to this laboratory by the Bureau of Standards of Washington, and has not been annealed [2]). Diameter 0.5 mm. Resistance at 0° C 0.064 Ω. Current and potential wires as under *a*.

β. a wire made of gallium, kindly furnished to this laboratory in 1923 by Professor F. M. JAEGER [3]). Diameter about 0.5 mm. Current and potential wires of copper melted into the gallium. Resistance at 0° C 0.167 Ω.

The pressure of the helium bath in these measurements was read both on a mercury manometer and a McLeod. The pressure was reduced by means of the mechanical pumps of the cryogenic laboratory. It appeared not to be possible with them to reduce the pressure so far that the gallium became supraconductive.

The earth magnetic field was not compensated.

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1) W. J. DE HAAS and J. VOOGD. These Proceedings **32**, 733, 1929. Comm. Leiden N⁰. 199*d*.

2) Cf. W. TUYN et H. KAMERLINGH ONNES, Arch. Néerl. (III A) **10**, 5, 1926. Comm. Leiden N⁰. 181, § 16.

3) Cf. W. TUYN et H. KAMERLINGH ONNES, l. c. § 14.

TABLE I.

Aluminium H. and B.		
p_{He} mm. Hg.	T °K.	Resistance Ω
790.4	4.26_2	1.4724
476.5	3.74_9	1.4721_5
158.5	2.89_5	1.4723
3.3_5	1.44_5	1.4723_5
1.0_5	1.23_8	1.4717
0.6_5	1.17_5	1.4656
0.5_5	1.15_7	1.4601
0.4_2	1.12_1	1.4402_5
0.3_8	1.10_8	1.4062
—	—	0.2884
0.3_4	1.08_7	0.0000
Measuring current 0.416 mA. $R_4/R_{273} = 0.073$.		

TABLE II.

p_{He} mm. Hg.	T °K.	Al—1924—II		Gallium
		Measuring current 42.4 mA. Ω	Measuring current 18.4 mA. Ω	Measuring current 42.4 mA. Ω
773.22	4.24	0.002499	—	0.000256_5
400.74	3.59	0.002497	—	0.000254
2.15	1.36_5	0.002502	—	0.000249
0.67	1.18_3	0.002498	—	0.000252_5
0.53_5	1.15_2	0.002472	0.002470	0.000254
0.49_5	1.14_3	0.002403	0.002353	—
0.48_2	1.14	variable	0.000000	—
0.42	1.11_9	0.000000	—	0.000250_5
0.31	1.08_3	0.000000	—	0.000250_5
R_4/R_{273}: for aluminium: 0.039, for gallium: 0.0015.				

In Fig. 1 the results of the measurements on Al-1924-II are represented.

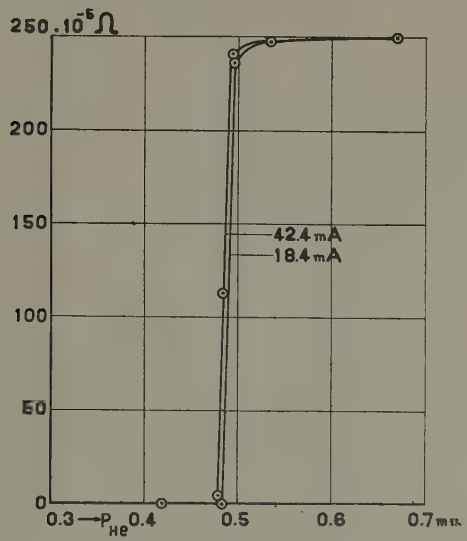

Fig. 1.

§ 3. *Conclusion.* From the fact that the resistance of Al-1924-II disappears within a temperature interval of less than 0.02 degree, we may safely conclude [1]) that pure aluminium becomes supraconductive [2]).

This conclusion is corroborated by the relatively small current sensitiveness of the resistance in the transition range. As a matter of fact this current sensitiveness corresponds to a change of the magnetic threshold value with temperature of the order of 175 gauss per degree.

We put the transition point of aluminium at the helium vapour pressure 0.49 mm, to which corresponds the temperature 1.14° K.

Table II confirms that the transition point of aluminium is decidedly higher than that of gallium.

I gladly express my thanks to C. J. MATTHIJS, phil. nat. cand., for his help with the measurements.

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[1]) Cf. W. J. DE HAAS and J. VOOGD. These Proceedings **34**, 192, 1931. Comm. Leiden N⁰. 214c, § 1.

[2]) By this the presumption expressed by J. A. M. VAN LIEMPT, Rec. Trav. Chim. **51**, 1117, 1932, regarding aluminium, is confirmed.

Physics. — *The Excitation of Band Systems.* I. By L. S. ORNSTEIN and G. O. LANGSTROTH. (Communication from the Physical Institute of the University of Utrecht.)

(Communicated at the meeting of April 29, 1933).

The relative transition probabilities for bands having the same electronic transition can be determined from the FRANK-CONDON theory, and the results are in semi-quantitative agreement with experiment in many cases. Such calculations are made possible by the fact that the wave functions for a molecule can be approximately written as the product of two functions, one depending on the electronic coordinates, and the other on the nuclear coordinates [1]). Therefore it is possible to separate the electric moment for any transition into a part dealing with the electronic transition and one dealing with the nuclear vibrations [2]). Hence in discussing the relative transition probabilities of bands of this type it is sufficient to consider only the nuclear motions, since the electronic transition is the same for all.

For a complete theory of the relative *intensities*, however, it is necessary to know the details of the accompanying phenomenon of excitation. The excitation probabilities for bands from different electronic levels is, of course, extremely complicated, but the excitation analogue of the FRANK-CONDON principle presents an obvious simplification. A beginning of the study of this problem is made in the present paper. Its final object is to make possible the formulation of a theory of excitation for band systems, starting with assumptions similar to those of the FRANK-CONDON theory for emission.

The fundamental point in the FRANK-CONDON classical picture [3]), which is upheld in its essentials by wave mechanics, is that during an electron transition the nuclear motions are unaffected, i.e. the Hamiltonian for the oscillating system at the end of the transition can be obtained from that at the beginning of the transition simply by replacing the initial nuclear potential function by the appropriate potential function for the final state. This applies equally well to the transition probabilities in absorption spectra. In this case also the nuclear momenta and position may be considered as unchanged [4]) during the electronic transition, presumably

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[1]) BORN and OPPENHEIMER. Ann. Phy. **84**, 457, 1927.

[2]) CONDON. Phy. Rev. **32**, 858, 1928: also HUTCHINSON. Phy. Rev. **36**, 410, 1930.

[3]) CONDON. Phy. Rev. **28**, 1182, 1926,

[4]) The classical picture is over-precise in this point since it violates the uncertainty principle.

because of the extremely small momentum of the light quantum, and of the extremely short transition time.

The question now arises, "How will the nuclear motions be affected in similar transitions which are due to electron impact?". FRANK and JORDAN [1]) suggested that the same considerations should hold here also. In the case of nitrogen for example, each nucleus is about ten thousand times as massive as the exciting electron, and it is perhaps to be expected that the direct effect on the nuclear motions should be very small. If this is so the relative intensities of bands having the same electronic transition but different initial vibrational quantum numbers in emission, should be independent of the velocity of the exciting electrons [2]).

The relative intensities of the bands $0 \rightarrow 2$ (λ 3805), $1 \rightarrow 3$ (λ 3755), and $2 \rightarrow 4$ (λ 3710) of the second positive group of Nitrogen were determined at various electron voltages from 14 to 25 volts. Measurements were taken so that the 'total' excitation curves (i.e. curves dependent on the probability of electronic transition as well) were also obtained, but we are chiefly interested in the relative intensities of the bands. They were found to be strongly dependent on the velocity of the exciting electrons for voltages below 20 volts, but above this value they became constant [3]). On the other hand, the development of the rotational structure did not differ appreciably over the voltage range examined.

It is not our purpose in this preliminary paper to attempt a detailed explanation of this behaviour but merely to present some experimental facts which will be discussed in a later publication.

Experimental.

The type of tube used in this research has been fully described elsewhere [4]). It consisted of hot filament from which electrons fell through a known accelerating potential and passed through a 'grid' into the field free observation space. It was filled with purified nitrogen to one tenth of a millimetre of mercury pressure. In general, the tube current used was 0.30 milliamperes.

The spectrograph was a standard quartz HILGER E 1, with a dispersion of about 10 A°/mm. at λ 3800. It was used with a very wide slit (about 0.5 mm.) in order to blend the rotational structure completely. Quartz optics were used. The exposure time was 30 minutes.

Several exposures were taken on each plate with the same tube current

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[1]) "Anregung von Quantensprüngen durch Stösse". Springer, (Berlin), 1926.

[2]) Provided of course that the nuclear and electronic functions can still be treated as independent factors. As a matter of fact, HERZBERG (Zeit. Physik. **49**, 761, 1928) has shown that the relative intensities of the bands can be varied in electrodeless ring discharge, but the processes involved are so complicated that it is difficult to say anything definite concerning their nature.

[3]) Data since obtained show that they remain constant for considerably higher voltages.

[4]) LINDH, Zeit. Physik. **67**, 67, 1930.

and exposure time, but with different accelerating potentials. In this way each plate had one exposure at least within the voltage range of some other plate, so that the determination of the total excitation curve, as well as the relative intensities of the bands at each voltage, was possible.

The intensity-blackening scale was put on each plate by means of a standard quartz band lamp and step-slit. In order that the exposure times for the bands and for the intensity marks should be equal, it was necessary to use the diffuse reflection from a 'white' plane smoked with magnesium oxide. The diffuse reflection coefficient for this substance changes by only a very small fraction of one percent over the range λ 3700 to λ 3800 [1]). Accordingly no appreciable error was made in using the energy distribution calculated for direct radiation from the lamp itself.

A MOLL microphotometer was used to obtain the relative intensities of the bands from their maximum blackenings in the usual way [2]). The use of the maximum blackening values is not in itself objection free for bands, since they may differ in the spacing or development of the rotational structure, in which case such measurements lead to erroneous results for the total band energies. Accordingly the intensity curves for the bands were determined for several plates by means of the analysing apparatus [3]) designed by WOUDA, and the areas under them were measured with a planimeter. Each area was then taken as a measure of the total band intensity. This method must give objection free results, and in the present case the values so found agreed with those found from the maximum blackening measurements within the experimental error.

There were no interfering bands or lines, and the bands under consideration were so far separated that the tail of one did not reach the head of the next. There was no background. The measured intensities are good to about 8 percent for a single determination.

Results.

The results for the total excitation functions are given in table I. The intensity of $0 \rightarrow 2$ at 18.0 volts has been set arbitrarily at 100, and the other values follow from this. No ν^4 corrections have been made since they are unnecessary for our purpose in the present paper.

The ratios of the band intensities at various voltages are given in table II.

The results of both these tables have been plotted in figures 1 and 2. The average values for the maximum blackening and integration determinations have been used. The approximate excitation curve for the negative band λ 3914 included in figure 1, has been taken from LINDH's paper (loc. cit.).

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1) TAYLOR, Jour. Opt. Soc. Am., 21, 776, 1931.
2) ORNSTEIN, MOLL, and BURGER, "Objektive Spektralphotometrie" (Vieweg) 1932.
3) Zeit. Physik., 79, 511, 1932.

TABLE I.
Relative Intensities of the Bands at Various Voltages.

Accelerating Voltage	0 → 2		1 → 3		2 → 4	
	M.B.	Area	M.B.	Area	M.B.	Area
12.0	0	0	0	0	0	0
14.0	23	24	13	15	—	—
14.0	26	26	14	14	3.4	3.6
15.0	65	67	39	43	10.2	9.9
15.0	63	60	36	36	8.3	9.5
15.9*	74	—	58	—	15.0	—
16.0	95	95	70	74	19.0	19.0
16.0	95	95	68	66	19.0	19.0
18.0	100	95	79	83	26.0	25.0
18.1	97	—	83	—	25.0	—
20.0	84	84	70	73	22.0	24.0
20.0	80	—	76	—	23.0	—
20.0	85	—	77	—	24.0	—
22.2	79	—	69	—	19.0	—
25.3	70	—	65	—	20.0	—

M.B. and Area refer to the values of the intensities calculated from maximum blackening and integration measurements.

* This value lies somewhat far off the curve drawn through the other values. The discrepancy is probably due to a change in tube current or accelerating voltage for this exposure, but since such was not noted at the time the value has been included in the results.

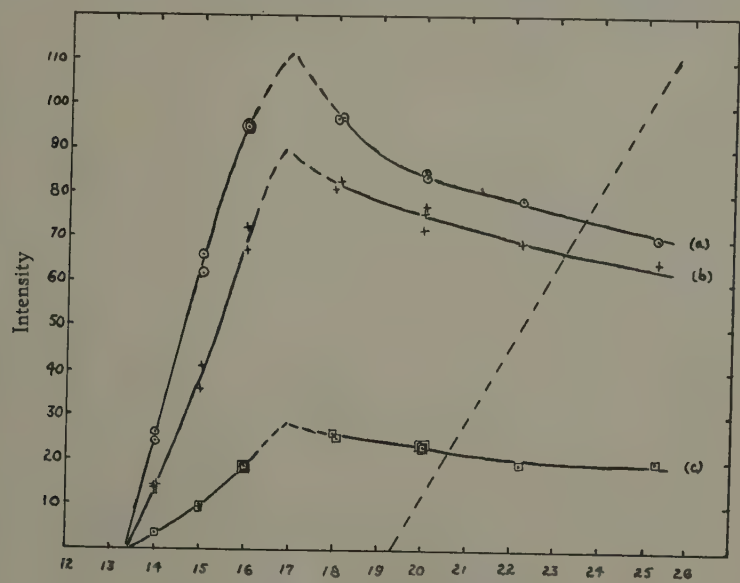

Fig. 1. Total excitation curves for three bands of the second positive group of Nitrogen. They are, in the order (*a*), (*b*), (*c*), 0 → 2, 1 → 3, and 2 → 4. The broken line represents approximately the excitation function for the negative band λ 3914.

TABLE II.
Band Ratios at Various Voltages.

Accelerating Voltage	1→3/0→2		2→4/0→2		2→4/1→3	
	M.B.	Area	M.B.	Area	M.B.	Area
14.0	0.57	0.62	—	—	—	—
14.0	0.53	0.57	0.13	0.14	0.24	0.26
15.0	0.60	0.65	0.16	0.15	0.26	0.23
15.0	0.57	0.61	0.13	0.16	0.23	0.26
15.9	0.78	—	0.20	—	0.26	—
16.0	0.74	0.78	0.20	0.20	0.27	0.26
16.0	0.72	0.70	0.21	0.20	0.28	0.29
18.0	0.79	0.88	0.26	0.27	0.33	0.30
18.1	0.86	—	0.26	—	0.30	—
20.0	0.84	0 88	0.26	0.29	0.31	0.33
20.0	0.94	—	0.28	—	0.30	—
20.0	0.91	—	0.28	—	0.31	—
22.2	0.87	—	0.24	—	0.28	—
25.1†	0.88	—	0.27	—	0.30	—
25.3†	0.87	—	0.26	—	0.30	—
25.3	0.93	—	0.28	—	0.31	—

† Values marked thus are not suitable for the determination of the "total" excitation curves (table I) because of slight variations in the tube current during an exposure. They may however be used in this table, since the ratios of the bands intensities at a particular voltage are independent of the current (within certain limits) for the magnitude of tube current used in this experiment.

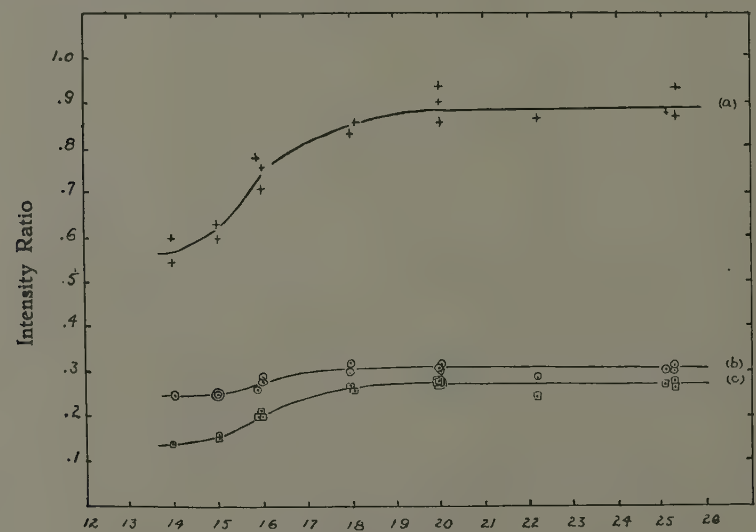

Fig. 2. Variation of the band ratios with the energy of the existing electrons. They are, in the order (*a*), (*b*), (*c*), 1→3/0→2, 2→4/1→3, and 1→3/0→2.

Discussion.

Inspection of figure 1 shows that the 'total' [1]) excitation curves have the usual form for triplet transitions. Extrapolation of the curves to zero intensity gives a value for the excitation potential of the second positive bands (13.4 volts) in good agreement with the value found by SPONER [2]) (13.0 ± .3 volts).

The curves for the variation of the intensity ratio of the bands with the energy of the exciting electrons (figure 2) show decided departures, below 20 volts, from straight lines parallel to the x axis. Above 20 volts however the intensity ratios are constant [3]). The fact that this constancy first appears near the point at which ionization sets in (16.5 volts) might be taken to indicate that it is due to the filling of the initial (emission) levels by the recombination of the molecular ion with an electron. This process would be, insofar as the relative populations of the initial levels are concerned, independent of the velocity of the exciting electrons. That this cannot be the explanation is shown by the following consideration.

If the constancy of the ratios is a result of recombination, it follows that above the voltage at which the constancy first occurs recombination must predominate in filling the initial levels. Now it is known from the excitation function for the negative band λ 3914 [4]) and from other sources, that the number of molecular ions and ions increases strongly with the voltage of the exciting electrons (cf. figure 1). We should then expect the intensities of the positive bands to increase proportionately. This is not the case as shown by figure 1, and therefore we must attribute the character of the curves in figure 2 to an excitation effect.

The relative excitation functions for these bands are therefore not independent of the velocity of the exciting electrons below 20 volts energy, and it is necessary to consider factors which are unimportant in the analogous theory for radiation. Above 20 volts, however, these factors probably become unimportant in excitation also (at least for the particular case examined). Further experimental data are being obtained, and will be presented along with a discussion of the problem in a later publication.

Examination of table I shows that there is no similar effect of electron impact on the molecular rotation, for if this were so it would cause a difference in the relative development of the rotational structure of the bands, which would result in different values for the relative intensities as calculated from maximum blackening and from integration measurements.

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1) We use the term "total" to indicate that the probability of excitation of the electronic part of the molecule is included.

2) Zeit. Physik., **34**, 622, 1925.

3) Results obtained since this paper was composed show that they are constant to considerably higher voltages, and that the intensities of the bands continue to decrease.

4) LINDH, loc. cit.

Our thanks are due to Mr. H. BRINKMAN for discussions on the subject, and to Mr. G. G. ZAALBERG for assistance in carrying out the experimental work. One of us (G. O. L.) is indebted to the Royal Commission for the Exhibition of 1851, whose award made his stay in Utrecht possible.

Hydrodynamics. — *On the application of statistical mechanics to the theory of turbulent fluid motion.* V.[1]) By J. M. BURGERS. (Mededeeling N⁰. 26 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft).

(Communicated at the meeting of April 29, 1933).

4. *Application of the normal functions obtained from equation* (19) *to the reduction of the exponent occurring in the distribution function* (12).

We proceed with the investigation of the statistical distribution of the relative motion and shall introduce the development of the stream function ψ of an arbitrary mode of relative motion (which satisfies the boundary conditions) according to the system of normal functions deduced from eq. (19).

It must be borne in mind that with the normalizing conditions assumed in § 3 the characteristic values of the parameter Λ all will be positive, provided that α is positive. It is convenient to adhere to the restriction of α to positive values, and to the type of functions obtained in § 3, but in the development of the stream function also terms will occur in which the sign of α is reversed (that is to say terms representing functions which are the symmetrical ones — with respect to the line $x=0$ — of those obtained before). As moreover both types of terms may have arbitrary phases with respect to x, we must expect that any stream function ψ will be built up from an aggregate of terms of the form:

$$\left.\begin{array}{l} a\,(\chi_I \cos \alpha x + \chi_{II} \sin \alpha x) + b\,(-\chi_I \sin \alpha x + \chi_{II} \cos \alpha x) + \\ + c\,(\chi_I \cos \alpha x - \chi_{II} \sin \alpha x) + d\,(\chi_I \sin \alpha x + \chi_{II} \cos \alpha x) \end{array}\right\} \quad . \quad . \quad . \quad (25)$$

It is convenient to introduce complex quantities, and so we assume that the stream function ψ_m of the mode of relative motion numbered m can be represented by the expression:

$$\psi_m = \tfrac{1}{2} \sum_{\alpha k} \left\{ e^{-i\alpha x} (A_{\alpha k}\, \chi_{\alpha k} + B_{\alpha k}\, \bar{\chi}_{\alpha k}) + e^{i\alpha x} (\bar{A}_{\alpha k}\, \bar{\chi}_{\alpha k} + \bar{B}_{\alpha k}\, \chi_{\alpha k}) \right\} \quad . \quad . \quad (26)$$

where $A_{\alpha k} = A^{I}_{\alpha k} + i\, A^{II}_{\alpha k}$, $\bar{A}_{\alpha k} = A^{I}_{\alpha k} - i\, A^{II}_{\alpha k}$, etc. Any mode of relative motion is now specified by the values of the A's and B's, and in calculating statistical mean values the summation with respect to the number m (i.e. the summation over the "ξ-space") can be replaced by an inte-

[1]) Part IV has appeared in these Proceedings, **36**, p. 276, 1933.

gration over all the A's and B's [1]). — In formula (26) the normal functions have been denoted by $\chi_{\alpha k}$, etc. in order to mark their dependence upon the parameter α; likewise we write $\Lambda_{\alpha k}$ for the characteristic numbers corresponding to a given value of α. The summation with respect to α properly speaking must be replaced by an integration; for simplicity of writing we provisionally keep to the notation used above.

From (26) we deduce:

$$\left.\begin{aligned} -\overline{u'v'} = \frac{i}{4} \sum_{\alpha k l} \alpha \, [(A_{\alpha k} \overline{A}_{\alpha l} - \overline{B}_{\alpha k} B_{\alpha l}) (\chi'_{\alpha k} \overline{\chi}_{\alpha l} - \chi_{\alpha k} \overline{\chi}'_{\alpha l}) + \\ + A_{\alpha k} \overline{B}_{\alpha l} (\chi'_{\alpha k} \chi_{\alpha l} - \chi_{\alpha k} \chi'_{\alpha l}) - \overline{A}_{\alpha k} B_{\alpha l} (\overline{\chi}'_{\alpha k} \overline{\chi}_{\alpha l} - \overline{\chi}_{\alpha k} \overline{\chi}'_{\alpha l})] \end{aligned}\right\} \quad (27)$$

$$\left.\begin{aligned} \overline{z} = \frac{1}{2} \sum_{\alpha k l} [(A_{\alpha k} \overline{A}_{\alpha l} + \overline{B}_{\alpha k} B_{\alpha l}) (\chi''_{\alpha k} - \alpha^2 \chi_{\alpha k}) (\overline{\chi}''_{\alpha l} - \alpha^2 \overline{\chi}_{\alpha l}) + \\ + A_{\alpha k} \overline{B}_{\alpha l} (\chi''_{\alpha k} - \alpha^2 \chi_{\alpha k}) (\chi''_{\alpha l} - \alpha^2 \chi_{\alpha l}) + \\ + \overline{A}_{\alpha k} B_{\alpha l} (\overline{\chi}''_{\alpha k} - \alpha^2 \overline{\chi}_{\alpha k}) (\overline{\chi}''_{\alpha l} - \alpha^2 \overline{\chi}_{\alpha l})] \end{aligned}\right\} \quad \ldots \quad (28)$$

from which the following expressions for the integrals are obtained:

$$-\int dy \, \overline{u'v'} \frac{d\lambda}{dy} = \frac{1}{2R} \sum_{\alpha k} \alpha^3 (A_{\alpha k} \overline{A}_{\alpha k} - B_{\alpha k} \overline{B}_{\alpha k}) \quad \ldots \quad (29)$$

$$\int dy \, \overline{z} = \tfrac{1}{2} \sum_{\alpha k} \alpha^3 \Lambda_{\alpha k} (A_{\alpha k} \overline{A}_{\alpha k} + B_{\alpha k} \overline{B}_{\alpha k}) \quad \ldots \quad (30)$$

Here use is made of equations (21) — (24) and of two other orthogonality relations:

$$\int dy \frac{d\lambda}{dy} (\chi'_k \chi_l - \chi_k \chi'_l) = 0. \quad \ldots \ldots \quad (31)$$

$$\int dy \, (\chi''_k - \alpha^2 \chi_k)(\chi''_l - \alpha^2 \chi_l) = 0 \quad \ldots \ldots \quad (32)$$

which latter (together with the equations obtained by changing every χ into its conjugate complex) are valid for all values of k and l, the case $k = l$ included.

Consequently the exponent occurring in the distribution function (12) assumes the form:

$$-\frac{\beta}{2R} \sum_{\alpha k} \alpha^3 \{ A_{\alpha k} \overline{A}_{\alpha k} (\Lambda_{\alpha k} - 1) + B_{\alpha k} \overline{B}_{\alpha k} (\Lambda_{\alpha k} + 1) \} \quad \ldots \quad (33)$$

The distribution function can have a meaning only if it remains finite

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[1]) As the $A_{\alpha k}$ are complex quantities the integration with respect to $A_{\alpha k}$ in reality stands for an integration with respect to the two real variables $A^{I}_{\alpha k}$ and $A^{II}_{\alpha k}$. A similar remark applies to the $B_{\alpha k}$.

for all values (finite or infinite) of the variables. This requires that all characteristic numbers $\Lambda_{\alpha k}$ shall be greater than unity, and thus imposes a certain condition on the function λ. If the condition is fulfilled we can calculate the statistical mean value $\overline{f}$ of any quantity f depending on the A's and B's, by means of the formula:

$$\overline{f} = (\Sigma\, \overline{n_m}\, f_m)/(\Sigma\, \overline{n_m}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (34)$$

where — as mentioned before — the summation with respect to m can be replaced by an integration over all the A's and B's.

5. *Introduction of a special function λ.*

In the theories developed by PRANDTL and by VON KARMAN much attention is given to considerations of similarity, and one of the questions which have arisen in this connection is, whether in some region in the neighbourhood of the wall the relative motions for various distances from the wall may be considered as similar, the scale being proportional to the distance. It seems possible to introduce an analogous consideration into the problem we are treating here, and it may be asked whether there may exist a certain similarity amongst the normal functions $\chi_{\alpha k}$. As the scale in the direction of the coordinate x is determined by the parameter α, it may be asked if, for any given value of the number k, the functions $\chi_{\alpha k}$ might be functions of a single variable $\xi = \alpha\,(y+\frac{1}{2})$ (in the following lines we shall write η in stead of $y+\frac{1}{2}$ for the distance from the wall at $y=-\frac{1}{2}$). As the presence of the other wall at $y=+\frac{1}{2}$ disturbs the similarity, we shall provisionally assume that the other wall is situated at a very large distance, so that, if it may happen that we find functions $\chi_{\alpha k}$ which decrease sufficiently fastly for large values of ξ, the presence of this second wall may be without appreciable influence upon them.

If we consider χ as a function of ξ, the differential equation (19), after division by α^4, takes the form:

$$\frac{d^4\chi}{d\xi^4} - 2\,\frac{d^2\chi}{d\xi^2} + \chi - i\,R\,\Lambda\left(\frac{1}{\alpha^2}\,\frac{d\lambda}{d\eta}\,\frac{d\chi}{d\xi} + \frac{1}{2\,\alpha^3}\,\frac{d^2\lambda}{d\eta^2}\,\chi\right) = 0 \quad . \quad . \qquad (35)$$

It is easily seen that α and η will disappear as separate variables form this equation, if we assume that $d\lambda/d\eta$ is proportional to $1/\eta^2$. It is convenient to write:

$$\frac{d\lambda}{d\eta} = \frac{b}{R\eta^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (36)$$

The constant b must be positive. In fact the expression can be applied in the neighbourhood of the wall $\eta = 0$ $(y=-\frac{1}{2})$ only, and it is evident that the statistical mean value of $-\overline{u'\,v'}$ must be positive here. From

(12) it will be seen that a positive statistical mean value of this quantity is to be expected only if $d\lambda/d\eta$ is positive.

Integrating (36) with respect to η, we obtain:

$$\lambda = constant - b \,/\, R\eta \quad \ldots \ldots \quad (37)$$

This expression, however, will violate the condition $\lambda = 0$ for $\eta = 0$ ($y = -\frac{1}{2}$) we have assumed in § 2. In order to amend this point we must assume that for very small values of η, say for $\eta < \delta$, the function $d\lambda/d\eta$ deviates from the course indicated by (36). This assumption, of course, is at variance with the similarity hypothesis; however, if we might suppose that the distance δ is sufficiently small, the similarity could be preserved approximately for values of η sufficiently surpassing δ. Such a case does not seem improbable; in fact an assumption of the same kind must be introduced by PRANDTL, as otherwise it would not be possible to understand how the constant *a* in eq. (**) of § 1 could take a definite value. — In order to fix the ideas we might assume f.i. that for $\eta < \delta$ the function $d\lambda/d\eta$ has the constant value:

$$\frac{d\lambda}{d\eta} = \frac{b}{R\delta^2} \quad \ldots \ldots \quad (38)$$

For abbreviation we write $b\,\Lambda = p$. With ξ everywhere as the independent variable, and using primes, etc. to denote derivatives with respect to ξ, we now arrive at the following differential equations for the function χ:

(*a*) in the domain $\xi < \alpha\,\delta$:

$$\chi^{IV} - 2\,\chi'' + \chi - \frac{ip}{(\alpha\,\delta)^2}\,\chi' = 0 \quad \ldots \ldots \quad (39a)$$

(*b*) in the domain $\xi > \alpha\,\delta$:

$$\chi^{IV} - 2\,\chi'' + \chi - i\,p\left(\frac{\chi'}{\xi^2} - \frac{\chi}{\xi^3}\right) = 0 \quad \ldots \quad (39b)$$

Equation (39*a*) can be solved by means of functions of the type $e^{m\xi}$, *m* being one of the four roots of an equation of the fourth degree. Hence the general solution for χ in the domain $\xi < \alpha\,\delta$ is of the form:

$$\chi = \Sigma\, B_\nu\, e^{m_\nu \xi} \quad \ldots \ldots \quad (40)$$

Equation (39*b*) can be reduced to a hypergeometric equation, which will be investigated in the next §.

6. *Investigation of equation* (39*b*).

In (39*b*) we write $\chi = \xi\,\chi_0$, and multiply by ξ; this gives:

$$\xi^2\,(\chi_0^{IV} - 2\,\chi_0'' + \chi_0) + 4\,\xi(\chi_0''' - \chi_0') - i\,p\,\chi_0 = 0 \quad \ldots \quad (41)$$

This equation can be transformed by LAPLACE's method, if we put:

$$\chi_0 = \int d\xi \, e^{\xi\zeta} \, w(\zeta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

ζ being an auxiliary variable. Then w must satisfy the equation of the second order:

$$\frac{d^2 w}{d\zeta^2} + \frac{4\zeta}{\zeta^2-1}\frac{dw}{d\zeta} - \frac{ip\zeta}{(\zeta^2-1)^2} w = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

while besides it must be ensured that the value of the expression:

$$e^{\xi\zeta}(\zeta^2-1)^2\left(w\,\xi - \frac{dw}{d\zeta}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

shall be the same at both ends of the path of integration.

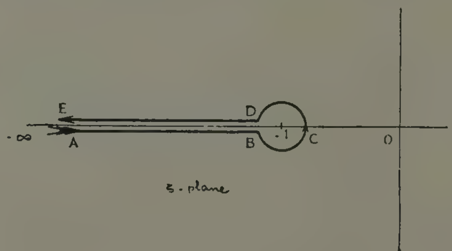
Equation (43) is a hypergeometric equation, having the singular points (all being regular):

$\zeta = -1$ with exponents: $\alpha_1 = \frac{1}{2}(-1-r+is)$, $\alpha_2 = \frac{1}{2}(-1+r-is)$
$\zeta = +1$ „ „ : $\beta_1 = \frac{1}{2}(-1-r-is)$, $\beta_2 = \frac{1}{2}(-1+r+is)$
$\zeta = \infty$ „ „ : $\gamma_1 = 3$, $\gamma_2 = 0$.

Here we have written $\sqrt{1+ip} = r+is$ etc.; p is a real and positive quantity, and we take r and s to be positive. — The function w thus can be defined by the scheme [1]:

$$w = P\left\{\begin{matrix} -1 & +1 & \infty & \\ \alpha_1 & \beta_1 & 3 & \zeta \\ \alpha_2 & \beta_2 & 0 & \end{matrix}\right\}$$

In order to obtain an integral for χ that does not become infinite for infinite values of ξ (which by nature is always real and positive), we must take the path of integration in such a way that the real part of ζ is always negative. This brings us to the path $ABCDE$; this



path at the same time ensures that the expression (44) shall vanish at both ends.

[1]) See f.i. E. T. WHITTAKER and G. N. WATSON, A course of modern analysis (Cambridge), § 10.7 and Chap. XIV.

In the vicinity of the point $\zeta = -1$ two linearly independent solutions of (43) are given by the following expressions, F being the symbol for the hypergeometric series:

$$w_1 = t^{\alpha_1} F(-1-r, -1+is, 1-r+is;\ t) \quad . \quad . \quad (45a)$$

$$w_2 = t^{\alpha_2} F(-1+r, -1-is, 1+r-is;\ t) \quad . \quad . \quad (45b)$$

Here t has been written for $(\zeta+1)/(\zeta-1)$; the series converge for all values of t satisfying $|t| \leqslant 1$, that is for all values of ζ having their real part negative or zero. For $|t| < 1$ the expression (45*a*) may be replaced by:

$$w_1 = t^{\alpha_1} (1-t)^3 F(2-r, 2+is, 1-r+is;\ t) \quad . \quad . \quad . \quad (45c)$$

It is also possible to write down two solutions, valid in the vicinity of $\zeta = \infty$; one of them is given by the series:

$$w_I = t^{\alpha_1} (1-t)^3 F(2-r, 2+is, 4;\ 1-t) \quad . \quad . \quad . \quad (46a)$$

while the other is of the form:

$$w_{II} = w_I \, lg\,(1-t) + c_0 + c_1 (1-t) + c_2 (1-t)^2 + \ldots \quad . \quad (46b)$$

These expressions are convergent for $|1-t| < 1$, the first one at any rate also for $|1-t| = 1$.

The functions w_1, w_2 are connected with the functions w_I, w_{II} by linear relations, which will be introduced subsequently. — In the special case when r is an integer $\geqslant 2$, the series defined by (45*a*) and (45*c*) and the one defined by (46*a*) break off, and reduce to polynomials. The function w_1 then is regular at $t = 1$ $(\zeta = \infty)$ and is equal to w_I multiplied by a constant factor.

Boundary conditions. — If we take w in the form $A_1 w_1 + A_2 w_2$, we obtain:

$$\chi = \xi \int d\zeta \, e^{\xi\zeta} (A_1 w_1 + A_2 w_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

There now are altogether six constants in our solution ($B_1, B_2, B_3, B_4, A_1, A_2$), which must be determined in such a way that $\chi = \chi' = 0$ for $\xi = 0$, while $\chi, \chi', \chi'', \chi'''$ must be continuous at $\xi = \alpha\delta$. Consequently there are also six homogeneous equations of the first degree for the six constants, and solutions different from zero can be obtained only if the determinant of the system vanishes. We shall not, however, try to develop an expression of this determinant for arbitrary values of δ, as this would require the evaluation of complicated integrals etc., but will turn at once to the case that δ becomes vanishingly small.

In that case the conditions $\chi = \chi' = 0$ for $\xi = 0$ can be applied at once to the expression (47), and lead to the equations: [1])

$$\lim_{\xi=0} \int d\zeta\, e^{\xi\zeta}\, \zeta (A_1\, w_1 + A_2\, w_2) = \mathit{finite} \quad . \quad . \quad . \quad . \quad (48)$$

$$\lim_{\xi=0} \int d\zeta\, e^{\xi\zeta} (A_1\, w_1 + A_2\, w_2) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

In the general case, r not being an integer, w_1 and w_2 assume constant values for $\zeta = -\infty$. If these values are denoted by a_1, a_2 respectively for the case $\arg(\zeta + 1) = -\pi$, $\arg t = 0$, which are the values that can be assigned to the arguments along the part $A\,B$ of the path of integration, then for the case $\arg(\zeta + 1) = +\pi$, $\arg t = 2\pi$ (which takes place along the part $D\,E$ of the path of integration) the limiting values become: $e^{2\pi i \alpha_1} a_1$, $e^{2\pi i \alpha_2} a_2$. Hence it will be seen that the condition (48) can be fulfilled only if:

$$A_1\, a_1 (1 - e^{2\pi i \alpha_1}) + A_2\, a_2 (1 - e^{2\pi i \alpha_2}) = 0 \quad . \quad . \quad . \quad . \quad (50)$$

It will appear from the results obtained below that this is also sufficient.

Investigation of the condition (49). — We begin with the parts of the integral relating to $A\,B$ and $D\,E$, which can be combined into the expression:

$$\lim_{\xi=0} \int_A^B d\zeta\, e^{\xi\zeta} [A_1\, w_1 (1 - e^{2\pi i \alpha_1}) + A_2\, w_2 (1 - e^{2\pi i \alpha_2})] \quad . \quad . \quad (51)$$

to be taken, as indicated, along $A\,B$.

We now put:

$$w_1 = a\, w_I + b\, w_{II}, \quad w_2 = c\, w_I + d\, w_{II} \quad . \quad . \quad . \quad . \quad (52)$$

the arguments of $\zeta + 1$ and of t in the points of $A\,B$ being as defined above. Making $\zeta = -\infty$ ($t = 1$), and having regard to the expressions

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[1]) Objections perhaps might be raised against the procedure of applying the conditions $\chi(0) = \chi'(0) = 0$ to the expression (47), as the point $\xi = 0$ is a singular point of eq. (39*b*). The same results, however, can be obtained in the following way: A finite value of σ is taken, and the equations expressing the continuity of $\chi, \chi', \chi'', \chi'''$ at $\xi = \alpha\sigma$ are written out in full. Then the exponential functions occurring in (40) are developed according to powers of σ: certain combinations of terms obtained in this way cancel in consequence of the relations $\Sigma B_\nu = \Sigma m_\nu B_\nu = 0$, which must be fulfilled in order that (40) satisfies the conditions at $\xi = 0$. Then a comparison is made of the terms of the lowest orders in σ on both sides of the equations. If now σ is made to decrease to zero, it is found that independently of the values of the B_ν the system of equations leads to certain relations between A_1 and A_2, viz. to eq. (50), which is equivalent with (48), and to eq. (49).

It may be remarked that also in the case of a finite value of σ the system of normal functions obtained for the case $\sigma \rightarrow 0$ can be used for the reduction of the integrals occurring in the distribution function, though the formulae will differ slightly from those deduced in § 4. We hope to come back to this point in a future paper.

(46a), (46b), we find: $a_1 = b\,c_0$, $a_2 = d\,c_0$. Hence if the relations (52) are substituted into the integral (51), it is found that the terms depending on w_{II} cancel in consequence of (50). The integral thus contains w_I only, and as w_I is of the order ζ^{-3} for $\zeta \to \infty$, it is convergent also when ξ is replaced by zero. The integral takes the form:

$$A_1\,(a - c\,a_1/a_2)\,(1 - e^{2\pi i \alpha_1}) \int_A^B d\zeta\, w_I\,.$$

By comparing the values of the various solutions for $t \to 0$ and also for $t = 1$, we obtain: $a - c\,a_1/a_2 = 1/F_1$, where F_1 has been written for $F(2-r, 2+is, 4;\ 1)$. We further introduce the expression (46a) for w_I into the integral and put: $\tau = 1 - t = -2/(\zeta - 1)$, so that $d\zeta = 2\,d\tau/\tau^2$. In this way the integral to be evaluated becomes:

$$\frac{2A_1}{F_1}\,(1 - e^{2\pi i \alpha_1}) \int_0^{1-\sigma} d\tau\,(1-\tau)^{\alpha_1}\,\tau\; F(2-r, 2+is, 4;\, \tau)\quad . \quad . \quad (53)$$

σ being the value of t at the point B. Replacing the factor τ before the function F by $1 - (1-\tau)$, it is required to calculate [1]:

$$\int_0^{1-\sigma} d\tau\,(1-\tau)^{\alpha_1}\,F - \int_0^{1-\sigma} d\tau\,(1-\tau)^{\alpha_1+1}\,F.$$

We begin with the first integral. To abbreviate we write: $F = \Sigma f_h\,\tau^h$. Then we make use of the equation [2]:

$$I_h \equiv \int_0^{1-\sigma} d\tau\,(1-\tau)^{\alpha_1}\,\tau^h = \frac{\Gamma(\alpha_1+1)\,\Gamma(h+1)}{\Gamma(\alpha_1+h+2)} - \frac{1}{1-e^{2\pi i \alpha_1}} \int_K d\tau\,(1-\tau)^{\alpha_1}\,\tau^h,$$

where the circuit denoted by K is defined by the formula: $\tau = 1 - \sigma\, e^{i\theta}$, θ moving from 0 to 2π. We expand $(1 - \sigma\, e^{i\theta})^h$ according to powers of σ by means of the binomial theorem; in this expansion it is sufficient to retain such terms σ^n only, as leave the real part of $\alpha_1 + n = n - \frac{1}{2}(r+1) + \frac{1}{2}\,i\,s$ negative, as the other terms may be made arbitrarily small by taking σ sufficiently near to zero. In this way we find:

$$I_h = \frac{\Gamma(\alpha_1+1)\,\Gamma(h+1)}{\Gamma(\alpha_1+h+2)} - \sum_{n=0}^{n=m} \frac{(-1)^n\, h!\, \sigma^{\alpha_1+n+1}}{(\alpha_1+n+1)\, n!\,(h-n)!},$$

where m is the greatest integer contained in $\frac{1}{2}(r+1)$.

---

[1] For the evaluation of the integral (53) I am indebted to the very valuable help of Dr. S. C. VAN VEEN at Dordrecht, and it is a pleasure for me to express my gratitude towards him also at this place.

[2] This equation is obtained by a process similar to that used by WHITTAKER and WATSON, l.c., § 12.43.

The integral to be calculated now is given by the sum [1]: $\sum_h f_h I_h$. Taking the first term of I_h, it appears that the sum:

$$F^* = \sum_h f_h \frac{\Gamma(\alpha_1+1)\,\Gamma(h+1)}{\Gamma(\alpha_1+h+2)}$$

can be transformed in such a way, that it takes the form of a hypergeometric series in which the independent variable has the value unity, together with some additional terms; the series can be summed by a known formula, so that F^* can be reduced to a relatively simple expression. — On the other hand it can be shown that for any value of $n < r$:

$$\frac{(-1)^n\,\sigma^{\alpha_1+n+1}}{(\alpha_1+n+1)\,n!} \sum_h f_h \frac{h!}{(h-n)!} = F_1\, g_n \frac{\sigma^{\alpha_1+n+1}}{\alpha_1+n+1},$$

where the g_n are the coefficients of the hypergeometric series

$$F(2-r, 2+is, 1-r+is; x).$$

A similar process can be carried out with the second integral, and in this way it is found that the original integral occurring in (53) can be transformed into an expression of the form:

$$\int_0^{1-\sigma} d\tau\,(1-\tau)^{\alpha_1}\,\tau F = (F^* - F^{**}) - F_1 \sum_n g_n \left(\frac{\sigma^{\alpha_1+n+1}}{\alpha_1+n+1} - \frac{\sigma^{\alpha_1+n+2}}{\alpha_1+n+2}\right). \quad (54)$$

By means of some further transformations the difference $(F^* - F^{**})$ can be brought into the following form:

$$F^* - F^{**} = -\frac{24}{p^2}\left\{\frac{\cos\frac{\pi}{2}(r+is)}{\cos\frac{\pi}{2}(r-is)} + 1\right\} \quad . \quad . \quad . \quad . \quad (55)$$

It remains to consider the part of the integral (49) relating to the circuit BCD. Here we may put at once $\xi = 0$. Further, as the real part of α_2 is positive, the contribution of w_2 into the integral can be made as small as we please by diminishing σ; hence it is sufficient to consider: $A_1 \int d\zeta\, w_1$. For the function w_1 the expression (45c) will be taken; upon integrating by terms it is found that the same series appears as occurred in (54).

Having regard to the constant factors before the integrals, it is easily seen that upon adding together the various terms, these series cancel in

[1]) The series $F = \Sigma f_h \tau^h$ is uniformly convergent in the domain $0 \leq \tau \leq 1 - \sigma$ and so term by term integration is allowed.

the final result, and thus equation (49) takes the form [1]):

$$-\frac{48\,A_1}{p^2\,F_1}\left(1-e^{2\pi i\alpha_1}\right)\left\{\frac{\cos\frac{\pi}{2}(r+is)}{\cos\frac{\pi}{2}(r-is)}+1\right\}=0\quad .\ .\ .\quad (56)$$

Characteristic values of the parameter p. — The characteristic values of p are determined by the condition that the coefficient of A_1 in (56) shall vanish. This condition reduces to the equation [2]):

$$\cos\frac{\pi}{2}(r+is)+\cos\frac{\pi}{2}(r-is)=0,$$

having the roots: $r=1, 3, 5, \ldots$ or, in general: $r=2k+1$, k being an integer. As: $1+ip=(r+is)^2$, it is found that the values of p are given by the expression:

$$p=4\,(2k+1)\sqrt{k^2+k}.$$

The case $k=0$ $(r=1)$ must be excluded, as p must be greater than zero.

(To be continued).

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[1]) In connection with the expression obtained for I_h and with the calculation of $\Sigma\, f_h\, I_h$ it may be remarked that it has not been proved that the whole sum of the neglected terms vanishes for $\sigma \rightarrow 0$ in such a way that the summation with respect to h can be executed absolutely safely. However, as Dr. VAN VEEN has pointed out to me, any difficulties arising from this circumstance can be obviated, by considering first the integrals for the case that r is a positive number included between the limits ε_1 and $1-\varepsilon_2$ (ε_1 and ε_2 being arbitrary positive numbers $<1/2$). In that case the real parts of both α_1 and α_2 are >-1, and the integrals of both w_1 and w_2 along the circuit BCD can be discarded, and also the integral along the circuit K occurring in the expression for I_h, so that the additional terms in the result for I_h are got rid of. By means of the theory of analytic continuation it then can be shown that the final result obtained for the integral (49) remains valid for all cases provided $r>-2$.

[2]) The factor $1/F_1$ has the value: $\Gamma(2+r)\,\Gamma(2-is)\,/\,6\,\Gamma(r-is)$.

Astronomy. — *Mittlere Lichtkurven von langperiodischen Veränderlichen.* XIII. *R Arietis.* Von A. A. NIJLAND.

(Communicated at the meeting of April 29, 1933).

Instrumente: S und R. Die Beobachtungen wurden alle auf R reduziert: die Reduktion R—S beträgt —0^m.19. Spektrum M3e (*Harv. Ann.* **79**, 164).

Der Stern ist von Anfang April bis Anfang Juni nicht beobachtbar: die letzte Beobachtung im Frühjahr erhielt ich am 3. April, die erste Sommerbeobachtung am 5. Juni. Es konnten mehrere Minima und Maxima ent-

weder gar nicht, oder nur sehr ungenau festgelegt werden. Bringt man ein paar Fälle am Schluss oder am Anfang der Sichtbarkeitsperiode, wo der Stern nicht gesehen wurde, und die in der Fig. 1 mit v bezeichnet sind, in Abzug, so ist die Gesamtzahl der hier zu besprechenden Beobachtungen 729 (von 2416835 bis 2427154). Es wurden wieder, wie in allen früheren Mitteilungen, die in zwei Instrumenten angestellten Schätzungen nur einmal gezählt. Pro Monat ist die Zahl der gesicherten Beobachtungen etwas grösser als sonst (2.9).

Die Tabelle I gibt eine Übersicht der benutzten Vergleichsterne.

TABELLE I. Vergleichsterne.

	BD	Hagen	St.	HA 29	HA 37	HA 91	*H*
B	+ 24°.325	—	60.0	6ᵐ.75	—	7ᵐ.08 F5	7ᵐ.04:
A	+ 23.308	—	54.9	—	—	— F0	7.73
a	+ 23.304	3	49.2[5]	8.15	—	— K0	8.35
b	+ 23.306	4	46.8	8.52	8ᵐ.61	8.80 G5	8.63
G	+ 24.323	—	43.7	9.06	9.26	—	[8.97]
c	+ 24.334	5	40.3	9.50	9.47	—	9.34
K	+ 24.332	8	36.1[5]	9.97	9.92	—	[9.80]
d	+ 24.327	7	35.2	9.88	—	—	9.91
x	+ 24.331	9	33.4	10.02	10.03	—	10.06:
y	—	11	27.9	—	10.75	—	10.73:
e′	—	12	27.6	11.15	—	—	10.76:
e	—	10	27.2	11.15	—	Grenze	10.79
f	—	14	23.1	11.00	11.10	11.28	11.24
g	—	16	16.8	—	—	—	11.93
h	—	20	11.6	—	—	—	12.50
j	—	28	6.3	—	—	—	13.08
k	—	—	0.0	—	—	13.54	13.64:

Stern *B* kommt in der PD vor ($7^m.34$, W+ oder $7^m.01$ im System der HP). Die Sterne *x*, *y* und *e′* kamen nur in vereinzelten Fällen zur Verwendung. Die Identifizierung der schwächeren Sterne in HA 37 (S. 153) ist schwierig und sehr unsicher, da hier die Koordinaten leider nur in Zehnteln Zeitminuten und in ganzen Bogenminuten gegeben sind[1]). Stern *k* (+ 13^s, —0′.5) ist wahrscheinlich mit *t* in HA 37 identisch. Das Spek-

[1]) Proc. 35, 931, Zeile 15 v. u.: lies *Zeitminuten* anstatt *Bogensekunden.*

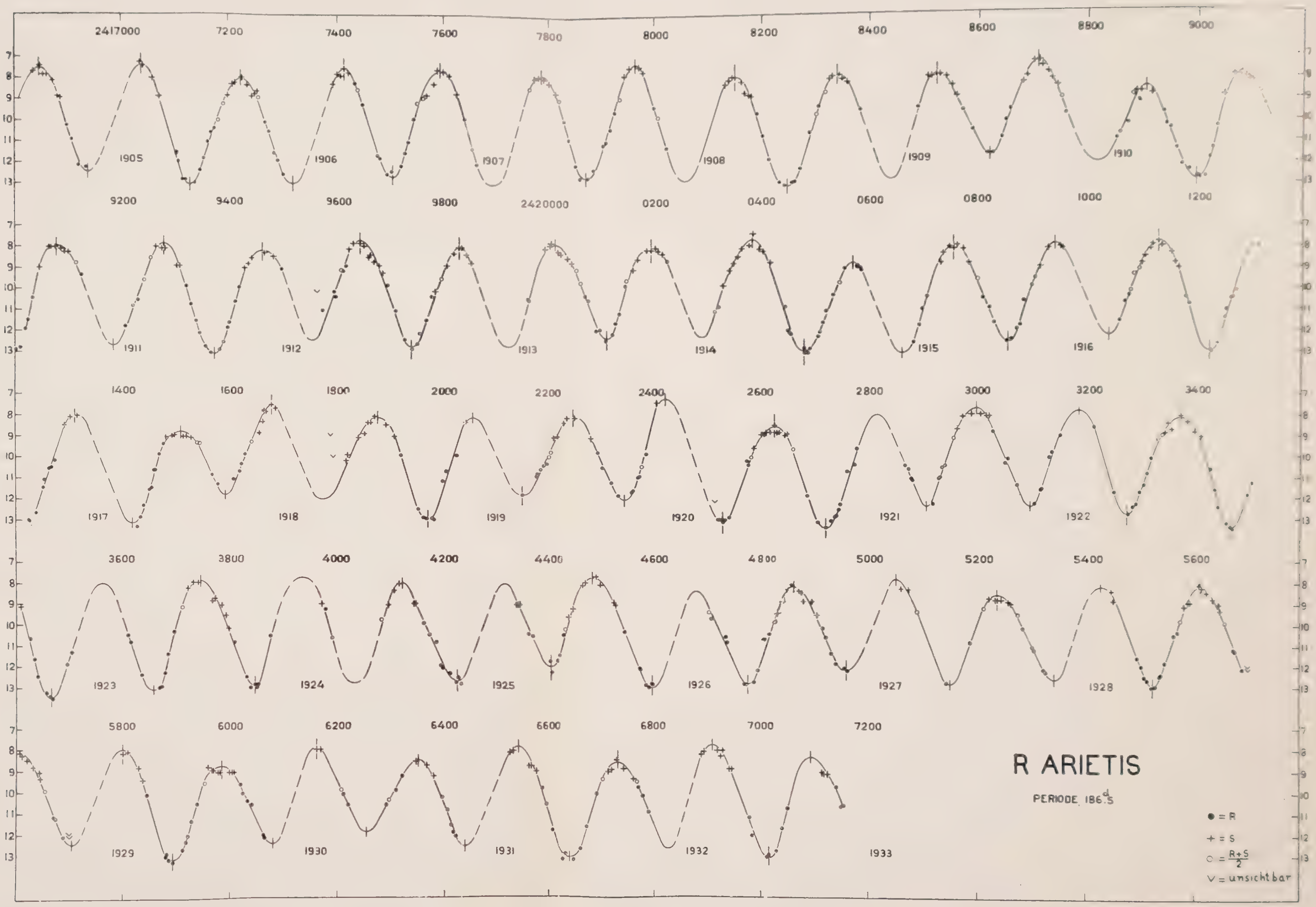

Fig. 1

trum der siebenten Spalte (Tabelle I) wurde den *Harv. Ann.* **91** entnommen. Zur Erzielung eines besseren Anschlusses an das System der HP wurden die Sterne *G* und *K* ein paar Mal mit beobachtet. Dieser Anschluss ist übrigens diesmal recht unbefriedigend. Zwischen den beiden *Harvard*-Katalogen **29** und **37** scheint ein systematischer Unterschied zu bestehen, der mich veranlasst hat, sämtlichen Grössen aus HA **29** eine Korrektion von —$0^{m}.41$ zu erteilen. Trotz dieser Korrektion weichen die Sterne *e* und *e'* noch stark ab. Stern *f* wurde viermal an die Grenze von *S*, Stern *k* dreimal an die Grenze von *R* angeschlossen. Die Stufenskala, die sich auf die Helligkeit $10^{m}.5$ bezieht, ist sowohl am oberen wie auch am unteren Ende etwas unsicher, da hier nur wenige Schätzungen vorliegen. Bei der Ableitung des Stufenwertes ($0^{m}.110$) haben die Sterne *B* und *k* tatsächlich nicht mitgestimmt.

Es liegen 78 Schätzungen der Farbe vor, welche aber für fünf Sechstel aus den Jahren 1905—1913 stammen. Aus den Tabellen II*a* und II*b* geht hervor, dass sich die Farbenauffassung weder mit der Zeit, noch mit der Helligkeit erheblich geändert hat. Das allgemeine Mittel ist $2^{c}.44$.

TABELLEN II*a* und II*b*. Farbenschätzungen.

Zeitraum	*n*	Farbe	Grösse	*n*	Farbe
241 7165—7598	21	$2^{c}.22$	$7^{m}.61$	13	$2^{c}.50$
7609—8327	22	2.23	7.93	13	2.47
242 8332—0023	22	2.59	8.04	13	2.61
0755—6930	13	2.88	8.14	13	2.35
	78		8.46	13	2.15
			9.23	13	2.54
				78	2.44

Die Figur 1 enthält die Beobachtungen, alle auf *R* reduziert. Die Reihe der Abweichungen (Beobachtung minus Kurve) zeigt 256 Plus-, 250 Minuszeichen, 223 Nullwerte, 232 Zeichenfolgen, 273 Zeichenwechsel. Das Mittel der absoluten Werte der Abweichungen ist $0^{m}.114$.

Ein Einfluss des Mondscheines auf die Helligkeitsschätzung ist nicht bemerkbar. Es verteilen sich auf 257 bei Mondschein angestellte Beobachtungen die Abweichungen wie folgt: 90 Plus-, 79 Minuszeichen, 88 Nullwerte.

Die Tabelle III enthält die aus der Kurve abgelesenen Epochen der Minima *m* und der Maxima *M*, nebst der Vergleichung mit den einfachen Elementen *R*:

$$2421788^{d} + 186^{d}.5\ E \text{ (für die Minima)}$$

und

$$2421878 + 186\ .5\ E \text{ (für die Maxima).}$$

TABELLE III.

E	Minima m					Maxima M				
	B	v	R	$B-R$	$B-F$	B	v	R	$B-R$	$B-F$
— 27	—	—	—	—	—	^{241}6846 :	^{m}7.5 :	6842^5	+ 3^5 :	+ 2 :
— 26	^{241}6936 :	^{m}12.4 :	6939	— 3 :	— 5 :	7036 :	7.3	7029	+ 7 :	+ 4 :
— 25	7127	13.0	7125^5	+ 1^5	— 1	7223	8.0	7215^5	+ 7^5	+ 4
— 24	7319 :	—	7312	+ 7 :	+ 4 :	7413 :	7.5	7402	+11 :	+ 7 :
— 23	7504	12.7	7498^5	+ 5^5	+ 2	7593	7.7	7588^5	+ 4^5	0
— 22	—	—	—	—	—	7786 :	7.8	7775	+11 :	+ 6 :
— 21	7872	12.7	7871^5	+ 0^5	— 4	7964	7.3	7961^5	+ 2^5	— 3
— 20	—	—	—	—	—	8150 :	8.0	8148	+ 2 :	— 4 :
— 19	8247	13.1	8244^5	+ 2^5	— 2	8339	7.9	8334^5	+ 4^5	— 1
— 18	—	—	—	—	—	8523	7.9	8521	+ 2	— 3
— 17	8619	11.7	8617^5	+ 1^5	— 2	8708	7.3	8707^5	+ 0^5	— 4
— 16	—	—	—	—	—	8904	8.6	8894	+10	+ 6
— 15	8995	12.9	8990^5	+ 4^5	+ 2	9078	7.9	9080^5	— 2^5	— 6
— 14	9183 ?	—	9177	+ 6 ?	+ 4 ?	9277	7.8	9267	+10	+ 7
— 13	9370	13.1	9363^5	+ 6^5	+ 6	9462	8.2	9453^5	+ 8^5	+ 7
— 12	—	—	—	—	—	9643	7.7	9640	+ 3	+ 2
— 11	9737	12.7	9736^5	+ 0^5	+ 2	9829	8.0	9826^5	+ 2^5	+ 2
— 10	—	—	—	—	—	^{242}0014	7.8	0013	+ 1	+ 2
— 9	^{242}0111	12.3	0109^5	+ 1^5	+ 4	0195	8.0	0199^5	— 4^5	— 3
— 8	—	—	—	—	—	0383	7.6	0386	— 3	0
— 7	0480	12.9	0482^5	— 2^5	+ 2	0569 :	8.7	0572^5	— 3^5 :	0 :
— 6	0661 ?	—	0669	— 8 ?	— 3 ?	0755	8.0	0759	— 4	0
— 5	0854	12.5	0855^5	— 1^5	+ 4	0937 :	7.9	0945^5	— 8^5 :	— 4 :
— 4	1036 ?	—	1042	— 6 ?	0 ?	1125	7.8	1132	— 7	— 2
— 3	1222	13.0	1228^5	— 6^5	0	1312 :	8.0	1318^5	— 6^5 :	— 1 :
— 2	1418 :	13.1 :	1415	+ 3 :	+ 9 :	1507	8.8	1505	+ 2	+ 7
— 1	1592 :	11.8 :	1601^5	— 9^5 :	— 3 :	1677 :	7.5	1691^5	—14^5 :	— 9 :
0	—	—	—	—	—	1876	8.1	1878	— 2	+ 3
+ 1	1970	12.9	1974^5	— 4^5	+ 1	2055 ?	—	2064^5	— 9^5 ?	— 5 ?
+ 2	2150 ?	11.8 :	2161	—11 ?	— 6 ?	2248	8.0	2251	— 3	+ 1

TABELLE III (*Fortsetzung*).

E	Minima *m*					Maxima *M*				
	B	*v*	*R*	*B—R*	*B—F*	*B*	*v*	*R*	*B—R*	*B—F*
+ 3	2344	12^m.0:	2347^5	— 3^5	+ 1	2421?	7^m.2	2437^5	—16^5?	—13?
+ 4	2529	13.0	2534	— 5	— 1	2626	8.5	2624	+ 2	+ 5
+ 5	2720	13.3	2720^5	— 0^5	+ 2	—	—	—	—	—
+ 6	2906:	12.3:	2907	— 1:	+ 1:	2997	7.6	2997	0	+ 1
+ 7	3094	12.3	3093^5	+ 0^5	+ 1	3182?	—	3183^5	— 1^5?	— 2?
+ 8	3272	12.7	3280	— 8	— 8	3368	8.2	3370	— 2	— 3
+ 9	3466	13.3	3466^5	— 0^5	— 1	—	—	-	—	—
+ 10	3657	13.1	3653	+ 4	+ 2	3744	7.9	3743	+ 1	— 2
+ 11	3847	12.8	3839^5	+ 7^5	+ 5	—	—	—	—	—
+ 12	—	—	—	—	—	4121	8.0	4116	+ 5	+ 1
+ 13	4224:	12.7	4212^5	+11^5:	+ 8:	—	—	—	—	—
+ 14	4406	12.0	4399	+ 7	+ 3	4484	7.7	4489	— 5	—10
+ 15	4597?	13.0	4585^5	+11^5?	+ 7?	—	—	—	—	—
+ 16	4777	12.8	4772	+ 5	+ 1	4861	8.1	4862	— 1	— 6
+ 17	4959:	12.2	4958^5	+ 0^5:	— 4:	5048?		5048^5	— 0^5?	— 6?
+ 18	5149:	12.8	5145	+ 4:	0:	5235	8.6	5235	0	— 5
+ 19	5338?	—	5231^5	+ 6^5?	+ 3?	5424?	—	5421^5	+ 2^5?	— 2?
+ 20	5519	12.9	5518	+ 1	— 2	5608	8.2	5608	0	— 4
+ 21	5703:	—	5704^5	— 1^5:	— 4:	5800:	8.0:	5794^5	+ 5^5:	+ 2:
+ 22	5893	13.2	5891	+ 2	0	5985	8.8	5981	+ 4	+ 1
+ 23	6080?	—	6077^5	+ 2^5?	+ 2?	6161?	7.9:	6167^5	— 6^5?	— 8?
+ 24	6253?	—	6264	—11?	—11?	6350	8.5	6354	— 4	— 5
+ 25	6438?	—	6450^5	—12^5?	—11?	6542:	7.8	6540^5	+ 1^5:	+ 1:
+ 26	6640	13.1	6637	+ 3	+ 5	6732	8.6	6727	+ 5	+ 6
+ 27	—	—	—	—	—	6911	7.8	6913^5	— 2^5	0
+ 28	7015	13.1	7010	+ 5	+ 9	7093?	—	7100	— 7?	— 4?
		12.70			± 4		7.96			± 4

Die übrigbleibenden *B—R* sind für diese glatte Kurve zu gross, und zeigen einen ausgeprägt systematischen Charakter; es wurde für Minima und Maxima zusammen auf graphischem Wege ein Sinusglied abgeleitet, und die definitiven Elemente *F* lauten dann:

$$\left.\begin{array}{l}\text{Minimum: } 2421787^{\mathrm{d}} \\ \text{Maximum: } 2421878\end{array}\right\} + 186^{\mathrm{d}}.5\, E + 5^{\mathrm{d}}.5 \sin 10^{\circ} (E - 7);$$

$$\frac{M-m}{P} = 0.488.$$

Auf eine genaue Rechnung habe ich verzichtet.

PRAGER's Katalog für 1933 gibt den Periodenwert 186$^{\mathrm{d}}$.41, und das aus sämtlichen von mir seit d. J. 1905 in den *Astr. Nachr.* mitgeteilten Epochen der Minima und Maxima abgeleitete allgemeine Mittel ist 186$^{\mathrm{d}}$.5. Die Hinzuziehung des Sinusgliedes drückt die Quadratsumme der Abweichungen für die Minima und die Maxima zusammen von 3267 auf 2045 herab. Für die älteren Epochen gelten scheinbar andere Elemente (s. *G. und L.* I, S. 53). Weder das von CHANDLER, noch das von MÜLLER abgeleitete Sinusglied (mit einer Periode von 72, bzw. 62 Perioden), das die älteren Beobachtungen sehr befriedigend darzustellen scheint, verträgt sich mit meinen Epochen.

Die extremen Werte des Lichtwechsels sind:

$$\left.\begin{array}{l}\text{Minimum: } v = 12^{\mathrm{m}}.70 \pm 0^{\mathrm{m}}.075 \\ \text{Maximum: } v = 7\phantom{^{\mathrm{m}}}.96 \pm 0\phantom{^{\mathrm{m}}}.056\end{array}\right\} \text{(m.F.)}.$$

Die Amplitude beträgt somit 4$^{\mathrm{m}}$.74. Beim Minimum scheinen die Abweichungen vom Mittelwert regellos aufzutreten, beim Maximum überwiegen die Zeichenwechsel stark. Bemerkenswert sind die hellen Minima 8619, 1590 und 2150, sowie das flache Maximum 1507.

Es wurde wieder der mittlere Verlauf der Lichtkurve in der Umgebung der beiden Hauptphasen durch Ablesung der Helligkeit für je 10$^{\mathrm{d}}$ abgeleitet. Von dieser Diskussion wurden die in der Tabelle III mit :, bzw. ? bezeichneten unsicheren Minima und Maxima ausgeschlossen.

Die beiden Teilkurven schliessen sich genau an einander an (s. die Fig. 2) und geben zusammen den Verlauf der mittleren Kurve. In dieser Kurve fällt das Minimum auf 1788, das Maximum auf 1879, also beide Hauptphasen einen Tag später als es die Elemente des Lichtwechsels verlangen.

In der Tabelle IV wurde 1788 als Nullpunkt der Phase genommen. Die

TABELLE IV. Die mittlere Kurve.

Phase	v	Phase	v	Phase	v	Phase	v
-50^{d}	$9^{\mathrm{m}}.97$	0^{d}	$12^{\mathrm{m}}.70$	$+50^{\mathrm{d}}$	$9^{\mathrm{m}}.58$	$+100^{\mathrm{d}}$	$8^{\mathrm{m}}.08$
-40	10.71	$+10$	12.54	$+60$	8.91	$+110$	8.38
-30	11.44	$+20$	11.93	$+70$	8.39	$+120$	8.86
-20	12.12	$+30$	11.14	$+80$	8.10	$+130$	9.52
-10	12.56	$+40$	10.34	$+90$	7.96	$+140$	10.22

Kurve verläuft glatt, wie es übrigens auch die Betrachtung der Fig. 1 lehrt, und zeichnet sich durch eine sehr kleine Schiefe aus.

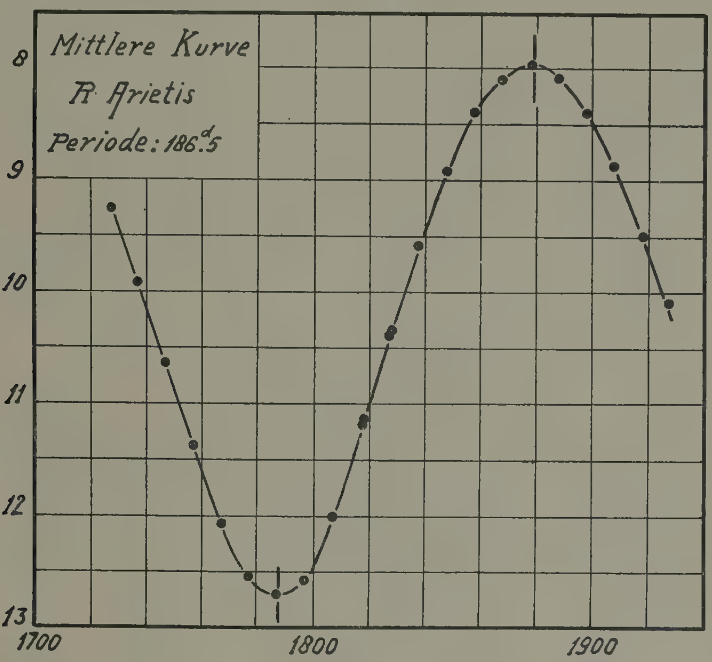

Fig. 2.

Die Streuung in der Nähe von 50^d erreicht die Werte:

	m	M	Mittel
im aufsteigenden Aste:	$0^m.274$	$0^m.314$	$0^m.294$
im absteigenden Aste:	$0\ .272$	$0\ .299$	$0\ .285$
Mittel:	$0\ .273$	$0\ .307$	

Die Streuung ist wieder grösser beim Maximum, und grösser im aufsteigenden Aste. Das Verhältnis der Streuungen $0^m.294$ und $0^m.285$ ist 1.03, das Verhältnis der durchschnittlichen Geschwindigkeiten des Lichtwechsels bei Auf- und Abstieg 1.05.

Zusammenfassung.

Aus 729 in den Jahren 1905 bis 1933 (2416835 bis 2427154) angestellten Beobachtungen von *R Arietis* sind die folgenden Elemente des Lichtwechsels abgeleitet worden:

$$\left.\begin{array}{ll}\text{Minimum:} & 2421787^d \\ \text{Maximum:} & 2421878\end{array}\right\} + 186^d.5\,E + 5^d.5\,\sin 10^\circ (E-7);\quad \begin{array}{l} v = 12^m.70 \\ v = 7.96 \end{array}$$

Amplitude $= 4.74$,

woraus $\dfrac{M-m}{P} = 0.488.$

Die mittlere Lichtkurve hat einen vollkommen glatten Verlauf.

Utrecht, April 1933.

Chemistry. — *On absorption and osmosis.* II. By F. A. H. SCHREINEMAKERS and C. L. DE VRIES.

(Communicated at the meeting of April 29, 1933).

I. *The osmotic system water* $+ Na_2CO_3$ *with a membrane of pig's bladder.*

We imagine in the osmotic system

$$L\,(W + X)\,|\,L'\,(W + X) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which $W =$ water and $X = Na_2CO_3$, a membrane of pig's bladder and the two liquids placed in such a way always that the left-side liquid always has a smaller X- and consequently a greater W-amount than the right-side liquid. As will appear from the experimental investigation to be discussed later on, the osmosis will then always proceed according to the D.T.

$$\leftarrow X \quad \rightarrow W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

no matter what concentrations the liquids L and L' may have.

As the substance X (viz. the Na_2CO_3) diffuses $\leftarrow$, namely from the liquid with the greater-towards that with the smaller X-amount, X always diffuses in congruent and positive direction; the same obtains for the water, which always diffuses $\rightarrow$, namely from the liquid with the greater-towards that with the smaller W-amount. As both substances now diffuse in congruent direction, (2), therefore, will represent the congruent D.T.

We shall now discuss some of the experimental investigations of system (1).

1. First we take the osmotic system

$$L\,(beg.\ \text{Water})\,|\,L'\,(beg.\ 11.831\ \%\ X) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which, at the beginning of the osmosis, there is pure water on the left side and on the right side a liquid, containing 11.831 % of Na_2CO_3.

The data for this system are found in table A. In the first column we find the numbers of the successive determinations, in the second column the time, namely the number of hours passed after the beginning of the osmosis; in the third column we find the X-amount of the liquid L and in the last column the X-amount of the liquid L'.

It appears from Nos 1 and 2 that in 11 hours the X-amount of liquid L has increased from 0 % to 0.424 % and of liquid L' decreased from 11.831 % to 10.518 %; the further determinations show that the X-amount

of liquid L continuously increases and that of liquid L' continuously decreases. After 339.3 hours the two liquids have almost the same X-amount (viz. 5.030 % and 5.061 %) and consequently the osmosis is practically over.

TABLE A. System (3).

N°.	t in hours	% X of liq. L	Diffused according to the determinations				% X of liq. L'
			of liq. L		of liq. L'		
			gr X	gr W	gr X	gr W	
1	0						11.831
			←	→	←	→	
2	11.0	0.424	1.629	26.817	1.624	26.659	10.518
3	23.5	0.935	1.596	24.777	1.588	24.830	9.423
4	35.8	1.470	1.897	20.697	1.337	20.047	8.452
5	54.1	2.265	1.558	22.992	1.533	22.798	7.461
6	80.2	3.340	1.615	22.449	1.584	22.452	6.570
7	121.7	4.494	1.304	18.087	1.366	18.121	5.890
8	192.0	4.857	0.847	11.595	0.885	11.490	5.290
9	339.3	5.030	0.367	4.730	0.338	4.633	5.061

Successive determinations of the quantity and the composition of liquid L enable us to deduce how much X and how much water has been taken in or given off by this liquid L between two successive determinations; we find these diffused quantities in the 4th and 5th columns (sub : of liquid L); the arrows indicate the direction in which these substances have diffused.

From Nos 1 and 2 it appears that in 11 hours after the beginning of the osmosis 1.629 grs. of X have diffused ← and 26.817 grs. of water →. From Nos 2 and 3 it appears that between these two determinations (consequently in 23.5—11 = 12.5 hours) 1.596 grs. of X have diffused ← and 24.777 grs. of W →. From this also follows that in 23.5 hours counting from the beginning of the osmosis 1.629 + 1.596 = 3.225 grs. of X had diffused ← and 26.817 + 24.777 = 51.594 grs. of water →. Further determinations show that during the entire osmosis the substance X has diffused ← and the water →, so that the osmosis has taken place according to the D.T. (2).

The quantities of X and W that have diffused may also be deduced from

the successive determinations of the quantity and composition of liquid L'; from the results of these determinations, to be found in the 6th and 7th columns, also follows that the osmosis has taken place according to the D.T. (2).

Comparing the quantities of X that have diffused in the 4th column with those of the 6th column, we note small differences; this obtains also for the quantities of water diffused in the 5th and 7th columns. If, however, we take the experimental difficulties of these determinations into consideration and the change in the absorption of X and W by the membrane, so that during the osmosis it can also take in or give off small quantities of X and W, then it is clear that small differences will occur always. This change in the absorption by the membrane results from the change in the concentrations of the two liquids, with which it is in contact during the osmosis; of course a change in the nature of the membrane can also play a part here (comp. Comm. I).

Besides system (3) discussed above, the three following systems

$$L\,(beg.\ 0.491\ \%\ X)\,|\,L'\,(beg.\ 4.678\ \%\ X)\ .\ .\ .\ .\ .\ (4)$$

$$L\,(beg.\ 3.912\ \%\ X)\,|\,L'\,(beg.\ 7.822\ \%\ X)\ .\ .\ .\ .\ .\ (5)$$

$$L\,(beg.\ 7.825\ \%\ X)\,|\,L'\,(beg.\ 11.883\ \%\ X)\ .\ .\ .\ .\ (6)$$

have also been examined. It appeared from the experimental investigations [1]) that here also during the entire osmosis the substance X and the water run through the membrane congruently and positively and consequently according to D.T. (2).

2. We now take the osmotic system

$$inv.\,(Water)\,|\,L'\,(beg.\ 11.878\ \%\ X)\ .\ .\ .\ .\ .\ .\ (7)$$

in which only one variable liquid, viz. L', containing 11.878 % of Na_2CO_3 at the beginning of the osmosis; the other liquid consists of pure water which during the osmosis is being renewed at short intervals, so that we may consider the state on the left side of the membrane as practically invariant.

The data for this system are found in table B, in which the 1st, 2nd, 3rd and last columns have the same meaning as in table A; as, however, the left-side liquid is invariant, its composition remains 0 % of X during the entire osmosis. From the last column it appears that the X-amount of the variable liquid decreases continuously and has come down to 0.081 % after 674.2 hours (or about 28 days).

As system (7) contains only one variable liquid, the quantities of X and W that have diffused and the direction in which they have passed through

[1]) C. L. DE VRIES, Dissertation, Leiden 1932, Tables XXVII, XXVIII and XXIX.

the membrane, can only be deduced from successive determinations of the quantity and composition of the variable liquid L'. We find these quantities diffused in the 4th and 5th columns; the arrows indicate the direction in which these quantities have diffused.

TABLE B. System (7).

N⁰.	t in hours	$^0/_0$ X of the inv. liq.	Diffused		$^0/_0$ X of the var. liq.
			gr X	gr W	
1	0	0	←	→	11.878
2	7.6	—	0.034	10.889	10.954
3	18.1	—	1.655	17.920	9.934
4	32.1	—	2.198	27.238	8.617
5	46.1	—	1.989	25.075	7.550
6	64.4	—	2.264	28.302	6.468
7	89.4	—	2.899	34.834	5.266
8	116.7	—	2.597	32.130	4.300
9	147.2	—	2.425	31.388	3.471
10	183.8	—	2.405	34.005	2.670
11	233.8	—	2.414	40.479	1.772
12	294.2	—	0.766	41.894	1.097
13	345.7	—	0.971	35.020	0.738
14	411.2	—	0.791	44.816	0.442
15	485.2	—	0.486	47.845	0.257
16	557.2	—	0.251	45.434	0.156
17	674.2	—	0.184	53.676	0 081

It appears from these determinations that during the entire osmosis the substance X has diffused ← and the water →; so both substances have continuously passed through the membrane according to the D.T. (2).

The osmotic system

$$inv.\,(Water)\,|\,L'\,(beg.\,2.964\,^0/_0\,X)\,.\quad.\quad.\quad.\quad.\quad.\quad(8)$$

was examined successively four times for reasons that will be entered into later on; it appears from the investigations [1]) that here also the osmosis continuously took place according to the D.T. (2).

[1]) C. L. DE VRIES, l.c. Table XXXII—XXXV.

In the osmotic system

$$inv.\,(Water)\,|\,L'\,(W+X)+solid\;Na_2\,CO_3\,.\,10\,H_2O \quad . \quad . \quad (9)$$

liquid L' is in equilibrium with the solid hydrate $Na_2CO_3 \,.\, 10\,H_2O$. During the osmosis this solid hydrate disappeared and liquid L', which at first had been saturated with the hydrate, became unsaturated. It appeared from the investigations [1]) that the Na_2CO_3 and the water had again passed through the membrane during this osmosis according to the D.T. (2).

In the osmotic system

$$inv.\,(Water)\,|\,solid\;Na_2\,CO_3\,.\,10\,H_2O\,. \quad . \quad . \quad . \quad . \quad (10)$$

only the solid hydrate $Na_2CO_3 \,.\, 10\,H_2O$ is present on the right side of the membrane. At an experimental investigation, in which 196.5 grs. of this solid hydrate were taken, all solid substance had disappeared after 108 hours and on the right side an unsaturated liquid had formed. Investigation [2]) showed that here also the osmosis had taken place according to D.T. (2).

3. We now take the osmotic system

$$L\,(beg.\;Water)\,|\,inv.\,L'\,(11.485\;{}^0/_0\;X)\,. \quad . \quad . \quad . \quad . \quad (11)$$

in which on the right side an invariant liquid is present now, containing 11.485 % of Na_2CO_3; on the left side is a variable liquid L, consisting at the beginning of the osmosis of pure water. The data for this system are found in table C, which has been arranged in a similar way as table B.

From this it appears that the X-amount of the variable liquid, consisting of pure water at the beginning of the osmosis, increased continuously and had run up to 11.203 % after 548.3 hours. It appears from the direction of the arrows in the 4th and 5th columns that also in this system the osmosis had taken place according to D.T. (2).

The same obtains, as is clear from our investigations [3]) also for the osmotic system

$$L\,(beg.\;Water)\,|\,inv.\,L'\,(W+X)+solid\;Na_2\,CO_3\,.\,10\,H_2O\,. \quad (12)$$

in which liquid L' was continuously kept saturated and consequently invariant by the presence of a sufficient quantity of solid $Na_2CO_3 \,.\, 10\,H_2O$.

II. *The sluice and the real diffusion-type.*

When we bring the membrane of the osmotic system

$$L\,(W+X)\,|\,L'\,(W+X)\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

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1) C. L. DE VRIES, l.c. pg. 53.
2) C. L. DE VRIES, l.c. pg. 54.
3) C. L. DE VRIES, l.c. Table XXVI.

TABLE C. System (11).

Nº.	t in hours	$^0/_0$ X of the var. liq.			$^0/_0$ X of the inv. liq.
			gr. X	gr W	
1	0	0	←	→	11.485
2	10.0	0.300	1.082	24.396	—
3	33.8	1.236	2.526	50.218	—
4	58.0	2.530	2.228	45.390	—
5	82.5	4.167	1.698	36.619	—
6	109.0	4.851	1.463	31.663	—
7	150.5	5.953	1.867	42.263	—
8	201.5	7.250	1.651	39.651	—
9	264.7	8.629	1.342	33.143	—
10	338.3	9.796	0.837	23.280	—
11	434.3	10.665	0.458	15.146	—
12	548.3	11.203	0.388	7.217	—

into liquid L, it will get a definite X- and W-amount in it, which we shall call x and w; if we bring this membrane into liquid L', then it will get an X- and a W-amount x' and w' in it.

If now $x < x'$ and $w > w'$, we represent system (13): by

$$L\,(W+X)\left|\begin{matrix} x < x' \\ w > w' \end{matrix}\right|L'\,(W+X) \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If we make a sluice-arrangement [1]) of this membrane, we see that the substance X will diffuse ← and the water →; consequently the sluice-D.T. of (13) is:

$$\leftarrow X \quad \rightarrow W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

If now not only $x < x'$ but also $w < w'$, we represent system (13) by:

$$L\,(W+X)\left|\begin{matrix} x < x' \\ w < w' \end{matrix}\right|L'\,(W+X) \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The sluice D.T. of this system then will be:

$$\leftarrow X \quad \leftarrow \mathrm{o} * W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

From this it appears that we may say: the substance X (the water)

[1]) F. A. H. SCHREINEMAKERS, These Proceedings **32**, 1152 (1929) and following communications. Recueil Trav. Chim. Pays-Bas **50**, 221 and 883 (1931).

diffuses in the sluice D.T. from the liquid in which the membrane would get the greater X-amount (W-amount), towards the liquid in which the membrane would get the smaller X-amount (W-amount).

From this it follows that the sluice D.T. of a system is being determined by the X- and W-amount the membrane would get in the liquids L and L'; so it depends on the X- and W-curve of the membrane, which, as we have seen previously (Comm. I, figs. 1, 2 and 3), can have different shapes.

In order to apply these considerations to an example, we shall in connection with our further discussions take the osmotic system:

$$inv.\,(Water)\,|\,L'\,(W+X) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

We now first imagine that the X- and W-curves of the membrane have a shape like those in fig. 1 (Comm. I), which we shall call diagram I. As the membrane in the water will always get a smaller X- and a greater W-amount than in liquid L', the sluice D.T. will be

$$\leftarrow X \quad \rightarrow W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

no matter what composition liquid L' may have.

If we now suppose in fig. 2 (Comm. I) an X-curve like the one in fig. 1 (Comm. I), we shall call this figure diagram II. If we now take liquid L' of system (18) somewhere between the points W and s, then the membrane will get a smaller W-amount in the water than in liquid L'. During the entire osmosis the sluice D.T. will now be:

$$\leftarrow X \quad \leftarrow o * W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

If, however, we take liquid L' somewhere between s and X, the membrane will get a greater W-amount than in liquid L'; we then once more get the sluice D.T. (19).

As, however, the X-amount of liquid L' decreases continuously during the osmosis and liquid L', therefore, moves towards point W along the line WX of the diagram, the sluice D.T. (19) will at a certain moment of the osmosis pass into (20) again.

Limiting ourselves to solutions of liquid L' of which the concentration is not too high, it follows:

$$\textit{sluice D. T. of diagram I} \quad \leftarrow X \quad \rightarrow W \quad . \quad . \quad . \quad . \quad (21)$$

$$\textit{sluice D. T. of diagram II} \quad \leftarrow X \quad \leftarrow o * W \quad . \quad . \quad . \quad (22)$$

If we leave an osmotic system alone, (i.e. if we do not make a sluice arrangement of the membrane), the substances will diffuse according to some D.T., which we shall call the real D.T. of this system. It is clear that during the osmosis a system can successively have different real D.T.'s also.

We now may put the question whether the real D.T. of a system is the

same or not the same as the sluice D.T. From theoretical considerations [1]) it follows that the two D.T.'s may be identical, but that this is not necessarily the case.

In order to illustrate this with some examples, we imagine in system (18):

$$X = \textit{tartaric acid, membrane of cellophane.} \quad . \quad . \quad . \quad (23)$$

From Chapter V (Comm. I) it now appears that the X- and W-curves of this system may be represented by diagram I, so that the sluice D.T. resembles that in (21); from the osmosis it appears that the real D.T. is identical [2]).

We now take in system (18)

$$X = \textit{tartaric acid, membrane of pig's bladder} \quad . \quad . \quad . \quad (24)$$

or

$$X = \textit{succinic acid, membrane of pig's bladder} \quad . \quad . \quad . \quad (25)$$

It now appears from Chapters III and IV (Comm. I) that the X- and W-curves of each of these two systems can be represented by diagram II, so that the sluice D.T. is the same as in (22). It appears from the osmosis that the real D.T. is identical.

We now take in system (18):

$$X = Na_2\,CO_3, \textit{ membrane of pig's bladder.} \quad . \quad . \quad . \quad (26)$$

From Chapter II (Comm. I) it now appears that it depends on the prehistory of the bladder whether the X- and W-curves can be represented either by diagram I or by II; so it also depends upon the prehistory of the bladder whether the sluice D.T. will be the same as in (21) or as in (22). From the experimental investigations of the osmosis of the systems (7), (8) and (9), of which (8) was determined four times successively, it appears that the real D.T. always is the same as in (21).

So here the real- and the sluice D.T. would be identical only then, when the X- and W-curves of the bladder used, belonged to diagram I. Previously, however, we have seen that a bladder belongs to diagram I only in very special cases, namely when it had already passed through some absorption-periods (Chapter II Comm. I). As it is not probable now that the various bladders, used for the determination of the osmosis, should accidentally all have belonged to diagram I, we may assume that. at least in some of these cases, the real- and the sluice D.T. will be different.

In order to make this still more probable, we determined, as has also been indicated in table D:

1°. the absorption of a bladder successively in liquids, containing 0, 1.5 and 3 % of Na_2CO_3.

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1) F. A. H. SCHREINEMAKERS, l.c.

2) F. A. H. SCHREINEMAKERS and J. P. WERRE, Osmosis in systems, consisting of water and tartaric acid. These Proceedings **34**, pg. 42, 162 and 477 (1932).

2°. afterwards with this same bladder the osmosis in system (8); this determination was repeated a second time.

TABLE D.

		Sluice D. T.	Real D. T.
1°.	Absorption	$\leftarrow X \leftarrow o * W$	
2°.	Osmosis		$\leftarrow X \rightarrow W$
3°.	Absorption	$\leftarrow X \leftarrow o * W$	
4°.	Osmosis		$\leftarrow X \rightarrow W$
5°.	Absorption	$\leftarrow X \leftarrow o * W$	
6°.	Osmosis		$\leftarrow X \rightarrow W$

3°. afterwards with this same bladder again the absorption as in 1°.
4°. afterwards again with the same bladder the osmosis in system (8).
5°. afterwards again with this same bladder the absorption as in 1°.
6°. and at last again with this same bladder the osmosis in system (8).

From these determinations [1]) it appeared, as has been indicated also in table D, that the sluice D.T. of this system was always

$$\leftarrow X \quad \leftarrow o * W$$

and the real D.T. always:

$$\leftarrow X \quad \rightarrow W$$

From this we may conclude that the sluice D.T. and the real D.T. of this system are different.

From this it appears that in the systems (23), (24) and (25) the sluice D.T. and the real D.T. are identical, but that these two D.T.'s differ in (26).

Leiden, Lab. of Inorganic Chemistry.

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1) C. L. DE VRIES, l.c. Table G and XXXII—XXXV.

Palæontology. — *The Shape and the Size of the Brain in* Sinanthropus *and in* Pithecanthropus. By EUG. DUBOIS.

(Communicated at the meeting of April 29, 1933).

[PLATES I—IV]

The most impressive if not the most significant of the numerous important human fossils unearthened in this still young century, was certainly the skull of *Sinanthropus pekinensis*, discovered on December 2, 1929 in the "Lower Cave" at Choukoutien, 40 km south-west of Peking, by the Chinese geologist W. C. PEI.

This nearly complete brain-case, only the base being deficient, and a second, much less complete one, discovered by PEI in the same cave, were prepared, studied and described by the author of the species name, DAVIDSON BLACK, Professor of Anatomy in the Peking Union Medical College. He published the results of his thorough investigation in two preliminary reports on the first and a notice on the second skull, in 1930, and finally in a splendid detailed craniometric and craniographic description of the external skull morphology of the two *Sinanthropus* specimens, including comparisons between these crania and those of other hominids and anthropoids, in 1931 [1]). He now concludes, as to the probable ontogenetic age and the sex of the two skull specimens, that the nearly complete brain-case, the Locus E skull, is that of an early adolescent. This conclusion is founded on the evidence of vigorous suture growth along the major vault sutures in this exceptionally thick-boned skull, not normally encountered in modern skulls of later adolescent age. The male sex is betrayed by the massively developed torus supraorbitalis and other male features, such as the strong contours of the zygomatic and supramastoid crests and the postero-inferior parietal thickenings in their relation to the torus occipitalis. The second, much less complete, the Locus D skull is considered to be that of a young adult, on account of the partial obliteration of the coronal suture. As a result of comparative study with other skulls the considered opinion is offered that, with a high degree of probability, the evident differences in form between the two skulls are due solely to differences in age and sex, the Locus D skull representing a female individual.

Judging from the preliminary reports, notice and photographs, which, in

[1]) DAVIDSON BLACK, On an Adolescent Skull of *Sinanthropus pekinensis* in comparison with an Adult Skull of the same Species and with other Hominid Skulls recent and fossil, Geological Survey of China. Palæontologia Sinica. Series D. Volume VII, Fascicle 2. Pp 1—144. With Plates I to XVI and Text figures 1 to 37. Peiping 1931.

1930, Professor DAVIDSON BLACK most kindly distributed to experts in the matter, both skulls appeared to be clearly neandertaloid, like the lower jaws and teeth of his first reports on the species. However, they also exhibited certain unique morphological features, which he regarded as evidences of archaic generalization. At the same time the first skull exhibited resemblances with *Pithecanthropus*, of which, still in his final report, he says: "it is clearly evident that the crania of *Sinanthropus* and *Pithecanthropus* resemble one another much more closely than they do any other known hominid type". Some other very able anatomists, WEIDENREICH, WEINERT, judging from those photographs, even found a striking similarity, which induced them unhesitatingly to include the fossil Peking Man in the genus *Pithecanthropus*, and to emphasize at the same time the close relation which, in their opinion, exists between the latter and Neandertal Man.

As to the probable taxonomic and phylogenetic status of *Sinanthropus*, DAVIDSON BLACK resumes his opinion as follows: — "Its cranial and dental characters are such as to imply that *Sinanthropus* could not have been far removed from the type of hominid from which evolved both the extinct Neanderthaloid and Rhodesian forms and the modern *Homo sapiens*". As I understand it, this is regarding *Sinanthropus* as the common ancestor of the two great hominid groups (species): *Homo neandertalensis* and *Homo sapiens*. His further study of certain of the major craniometric and craniographic characters which, to his mind, serve sharply to distinguish *Sinanthropus I* from other hominid types ancient or modern, and of the unique morphology of the tympanic portion of the temporal bone, which made it evident that the meatus and the middle ear in this adolescent *Sinanthropus* must have presented relations similar to those which in modern man are typically developed only in infants and very young children [1]), would seem amply to sustain this opinion.

Although acknowledging the resemblances exhibited by the crania of *Sinanthropus* and *Pithecanthropus*, it is equally apparent to DAVIDSON BLACK that they differ from one another in points of proportions and detail to a degree amply sufficient to proclaim their generic distinction. The strikingly higher and fuller frontal and parietal vault curve of the mid-sagittal contour of *Sinanthropus* constitutes a character serving sharply to distinguish this form from *Pithecanthropus*. Another important feature, in which these two crania, in norma basalis view, significantly differ is that the vertical planes in which fall the least frontal diameter and the greatest vault breadth, both occupy relatively and absolutely a position much further back upon the *Pithecanthropus* calvaria than on *Sinanthropus*. This is connected with a very striking and most significant difference to be observed in the outlines of the two mid-horizontal contours in glabella-opisthion orientation — norma basalis view. That of *Sinanthropus* is a long oval

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[1]) Further, the mammilary portion of the mastoid process is even more slightly developed than in Neandertal Man (of Krapina), recalling as it does the condition found in infants.

with full frontal and occipital arcs, not greatly dissimilar to that of Neandertal, and indeed many modern human skulls. The contour of *Pithecanthropus* on the other hand is significantly pyriform in outline with narrowed frontal and broad occipital arcs; the latter being quite markedly flattened posteriorly. The same conditions as in *Pithecanthropus* are found to obtain in no less marked degree in the juvenile crania of the great anthropoids — and, I may add, the adult crania of the gibbons.

To those differences between *Sinanthropus* and *Pithecanthropus*, so excellently described by DAVIDSON BLACK, as I could verify by comparing the calvaria of the latter with the now available beautiful cast made by Mr. F. O. BARLOW of the completely prepared *Sinanthropus* brain-case, there is no need to add any others, as they have no direct relation to differences in the shape of the brain, the proper subject of this report [1]).

The apparently archaic character of *Sinanthropus* appears to be in accordance with the geological age of the Choukoutien cave deposits, which, as judged from the mammalian fauna, is regarded as probably Lower Pleistocene (TEILHARD DE CHARDIN and YOUNG, PEI). However, there are in such estimations always elements of uncertainty. Possibly this age attributed to the fossil cave fauna may be subject to revision, since we now have conclusive, though indirect evidence of distinct humanity of *Sinanthropus*, in the established facts that he knew the use of fire and the manufacture of stone implements (PEY 1931), reminding of the Mousterian fashion according to Prof. BREUIL (1931), though representing a more crude lithic culture according to TEILHARD DE CHARDIN and PEI (1932).

Conclusive d i r e c t evidence as to complete humanity of *Sinanthropus* and his generic distinctness from *Pithecanthropus*, which was anticipated by the comparison of the crania, I expected from a comparison of the endocranial casts conveying the general shape and dimensions of the brain which filled the crania. Particularly interesting in this respect it would be to verify the significance of the strikingly higher parietal vault curve in the side view contour of *Sinanthropus* "constituting a character serving sharply to distinguish this form from *Pithecanthropus*", for indeed t h e p a r i e t a l v e r t e x o f t h e b r a i n i s a n e l e m e n t a r y d i s t i n c t i v e h u m a n c h a r a c t e r.

Many years ago, when I had entered upon studies on the cephalization of *Pithecanthropus*, I compared an endocranial cast of the latter with endocranial casts of Man on the one side, and Anthropoid Apes (Chimpanzee, Orangoutan, Gibbon) and some Monkeys (*Cebus*, *Midas*) having the highest "relative brain volumes", on the other hand, in order to find out a possible connection between relative volume and shape of the brain.

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[1]) One of these differences may, however, be noted here, as it sharply serves to distinguish *Sinanthropus* (and the Chimpanzee) from *Pithecanthropus* (and the Gibbons). This is the complete absence, on the nuchal part of the occipital bone, of the fossae for the attachment of the two rectus capitis posticus minor muscles, whose presence was so striking a feature in *Pithecanthropus*.

Comparison of the mid-sagittal outlines of telephotographic side views of those endocranial casts, in subcerebral (orbital—suboccipital) plane orientation, as represented in the two Figures of Plate IV (to be compared with the lower Figure of Plate III), did not prove the existence of such a connection. The outlines of *Pithecanthropus, Hylobates agilis* and Chimpanzee nearly agree, and in the parietal region, even those of the hypsicephalic orangoutan and also the Platyrrhine monkeys. Man, however, is distinguished from all of them by the possession of a high parietal vertex. At the same time his lobus temporalis, in distinction from all of them, appears as much more inclining to the front.

Doubtless this striking feature of the human brain-shape is to be considered as a consequence of this that in Man the head is poised upon the vertebral column, in distinction from the Apes, whose forward bent head is kept in position by the muscles attached at the nuchal part of the occipital bone. Apparently, in this respect *Pithecanthropus* was not humanlike.

Lately Prof. DAVIDSON BLACK supplied the fervently anticipated direct evidence of the perfectly human nature of *Sinanthropus*. In January his report on the endocranial cast of the Locus E skull, which cast he had made in 1930, was published [1]), and in the same month very exact copies of the original by Mr. F. O. BARLOW were made available, who most kindly sent the first finished one to the Teyler Museum, where it came at my disposal on February 8. From this date I have closely studied this extraordinarily interesting specimen, also by means of telephotographs (made with a lens of 150 cm focal length, set up at a distance of 3 meters from the object).

Plates I and II in this paper give reproductions of those telephotographs representing the norma verticalis and norma lateralis dextra views of this *Sinanthropus* endocranion, in $^3/_4$ natural size, beside the corresponding views, in the same reduced scale, of *Pithecanthropus*. The accurate comparative diagrammatic drawings of Plate III were made by the same and an analogous exact method.

My expectation of the significance of this endocranial cast had not been pitched too high. It really settled the question on the complete humanity of *Sinanthropus*.

It, moreover, gave evidence of an important peculiarity of this individual man. In his Report DAVIDSON BLACK says, that at first it had been intended to describe the endocranial cast of both this specimen and that of the adult skull, from Locus D, together, but as circumstances had retarded the work of restoration on the latter specimen, the report on the Locus E specimen would no longer be delayed. This means the joyful expectation of a fuller restoration of the other skull and the future availableness of another

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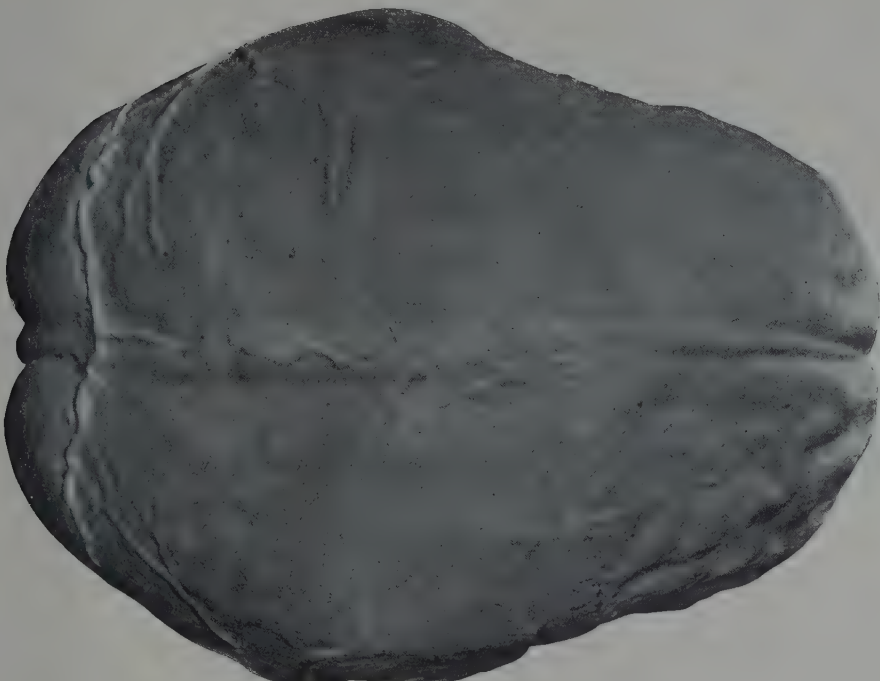
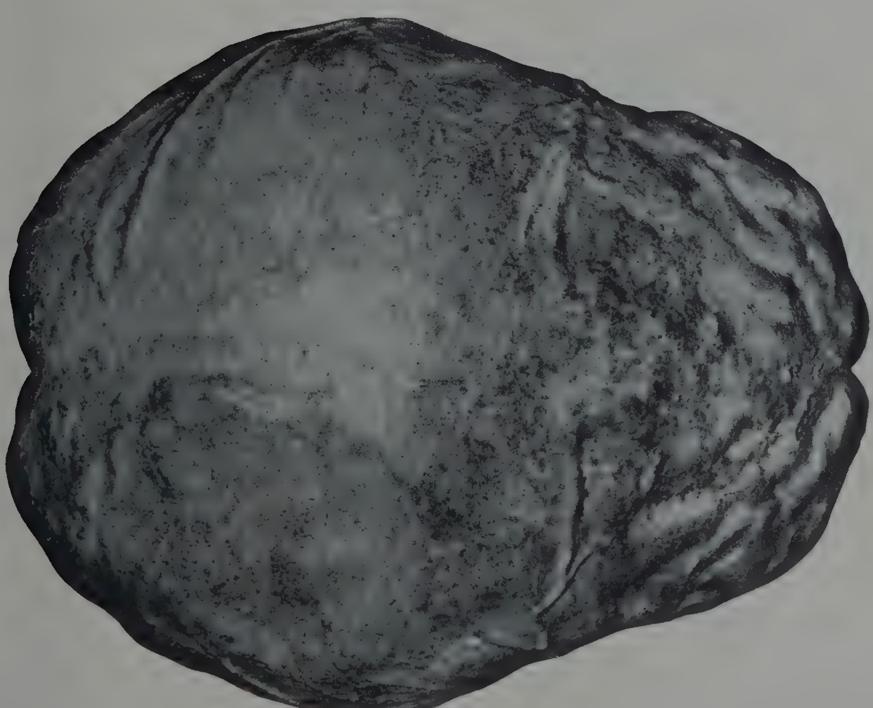
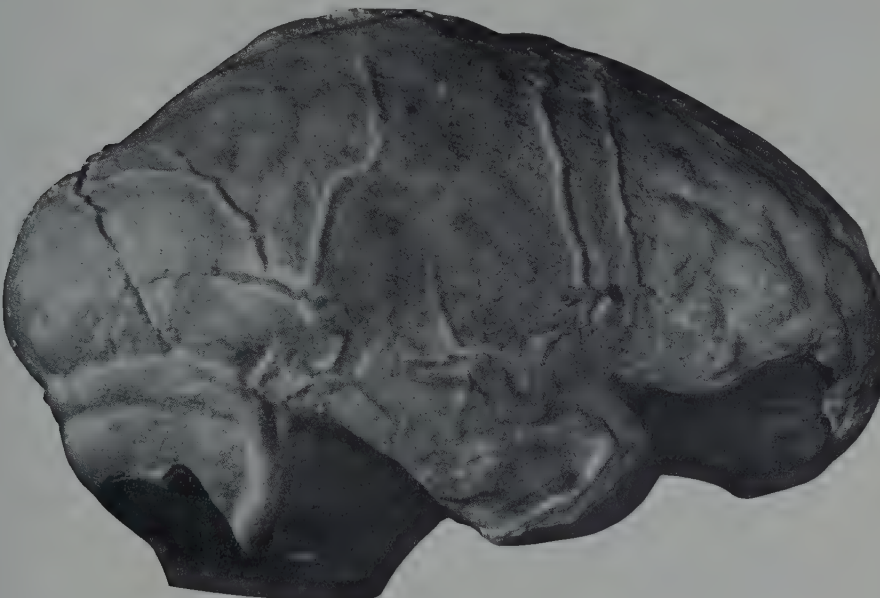
[1]) DAVIDSON BLACK, On the Endocranial Cast of the Adolescent *Sinanthropus* Skull. Proceedings of the Royal Society. Series B. Vol. 112. Biological Sciences. Pp. 263—276. With six Plates. London, January 2, 1933.

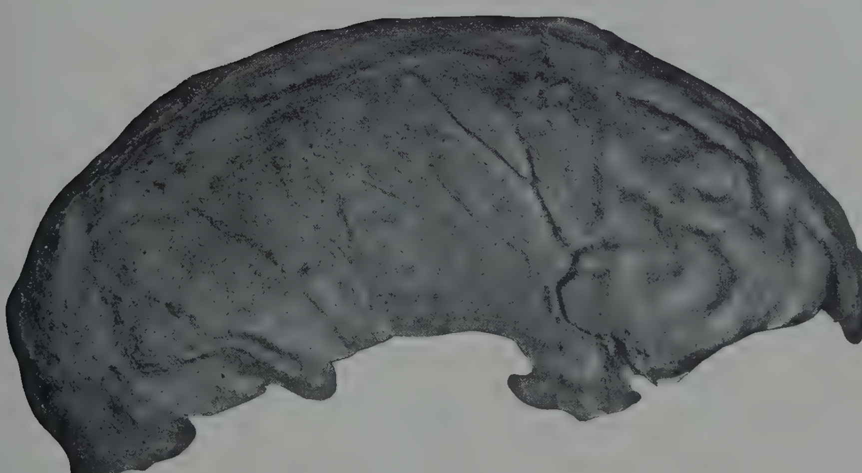
PLATE I.





Upper Figure: Endocranial cast of adolescent *Sinanthropus*.
Lower Figure: Endocranial cast of *Pithecanthropus erectus*.
Both telephotographs of norma verticalis view. $\times$ 3/4.





Upper Figure: Endocranial cast of adolescent *Sinanthropus*.
Lower Figure: Endocranial cast of *Pithecanthropus erectus*.
Both telephotographs of right norma lateralis view in subfrontal (orbital)-suboccipital cerebral plane orientation. $\times$ 3/4.

endocranial cast of the species, enabling us to verify the signification of that peculiarity of the locus E *Sinanthropus*.

In a most able manner DAVIDSON BLACK has restored the relatively small missing part of the first endocranial cast, apparently thus nearly approaching reality. This part having been painted an ochre colour is indicated in dark tone in the photographs.

Apparently the cast represented in the Plates of his report is the natural one, as appears from the irregularly interrupted vascular markings, the numerous diminutive pits, eminences and other small unevenesses on the surface of the preserved parts. These otherwise insignificant defects were obviously mended in the cast from which our copy was taken.

From comparative measurements of mean lengths in DAVIDSON BLACK's norma verticalis, frontalis and lateralis views, in photographs and diagrams of the endocranion report, our copy appears to be admirably exact in breadth and height, only slightly expanded in length, in proportion of 157 to 159 mm. Equal lengthening was found by comparing with the "inner skull length" in the skull report.

Having in this way, as much as possible, made sure of the exactness of the copy endocranial cast, I may now again draw the attention to the Plates I and II of this report, and proceed to compare *Sinanthropus* with *Pithecanthropus*.

There is obviously little difference in size between the two brains, in **shape**, however, they are surprisingly unlike.

As we could expect from the comparison of the skulls in the norma verticalis view, the brain form of *Sinanthropus*, in this view, is oblong and narrow against the more rounded, broad form of *Pithecanthropus* (Plate I).

Most strikingly different are the two brains in the norma lateralis view (Plate II). The frontal part in *Sinanthropus* is much fuller and rounder, as was anticipated from the shape of the brain-case. Likewise the dissimilarity in the shape of the parietal contour of the brain-case is repeated, but much more strikingly and significantly, on the brain itself. The contour line between the bregma and the lambda (in a straight distance of about 90 mm in both specimens), which is nearly flat in *Pithecanthropus*, is strongly elevated in *Sinanthropus* to a real and distinctly human parietal vertex. At the same time we find, herewith again in correlation, the temporal lobe humanly declined to the front.

These features are clearly demonstrated in Plate III, showing diagrammatic drawings of the norma lateralis view contour of the *Sinanthropus* endocranial cast in comparison with endocranial casts from a normal mesocephalic human (Dutch) skull and (in the inset) a microcephalic skull of 375 c.c. capacity [1]). Microcephaly, although greatly altering the brain shape in other respects, does not at all take away the parietal

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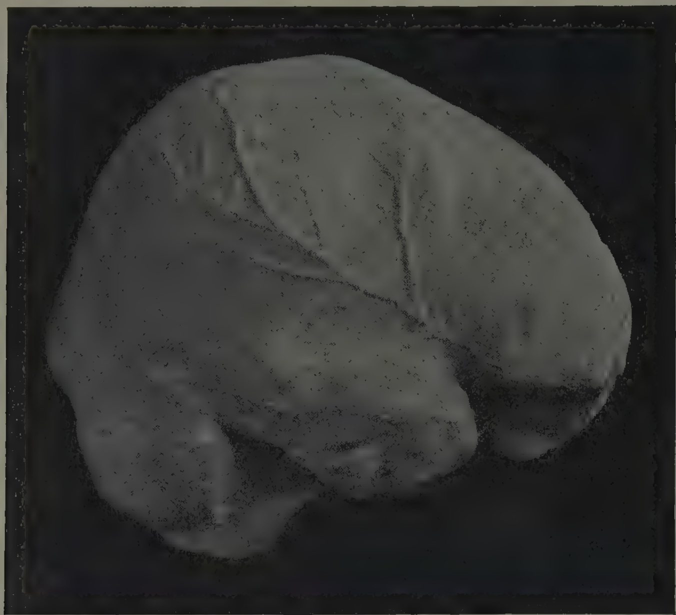
1) This is the skull in the Leiden Institute of Anatomy, described by SANDIFORT (1835).

vertex, that essential human character, which *Pithecanthropus* as well as the Apes does not possess (Plate III, lower Figure).

From this and other comparisons it appears to be a consequence of the oblong, dolichocephalic (cranial index 71) brain form of *Sinanthropus,* that its parietal elevation does not rise to quite such a height as in that mesocephalic brain, and that it is surpassed still more by the brachycephalic Javanese brain, as shown in the Text Figure on this page. Telephotograph of endocranial cast of Javanese skull, norma lateralis view in subcerebral (orbital-suboccipital) plane orientation. × ½.



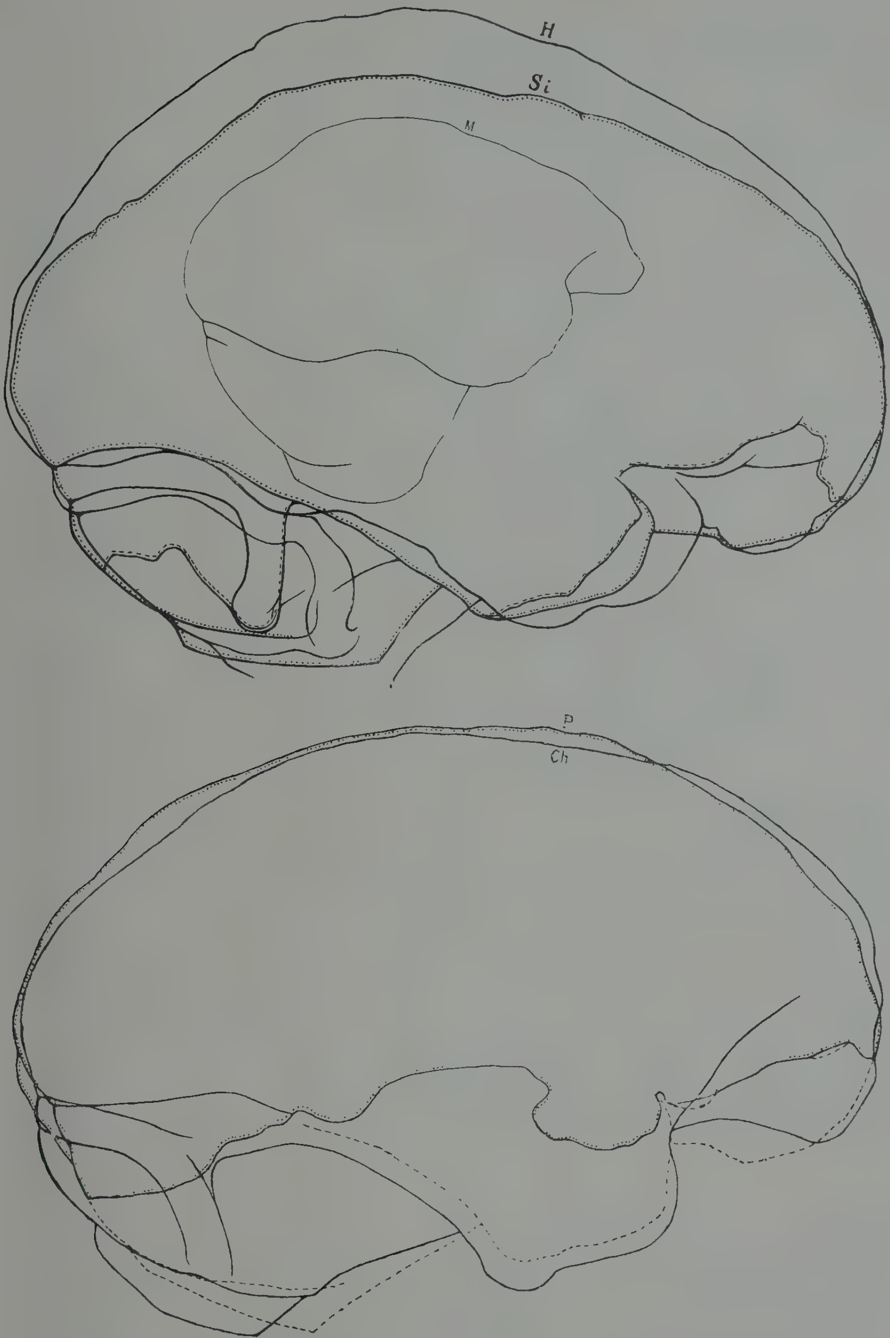
On the other hand, in the lower Figure of Plate III, the outlines of braincasts, in norma lateralis views, of *Pithecanthropus* and Chimpanzee [1]), which nearly coincide, lack a parietal vertex. The conclusion is obvious, that in *Pithecanthropus,* in distinction to *Sinanthropus,* the head was not poised on the vertebral column.

Another important character of this *Sinanthropus,* of which the endocranial cast, corresponding in shape and volume with the cranial cavity, may also give evidence, is the size of the brain.

Before knowing DAVIDSON BLACK's report on the endocranial cast, wherein he published the results of his accurate volumetric determinations, I had measured the volume of the copy of that cast, by means of a very exact hydrostatic balance, as to be 930 c.c. Apparently, however, this result of the most exact volumetric method, needed a correction, a slight

¹) Endocranial cast of a very fine adult female skull, capacity 385, from the Amsterdam Institute of Anatomy.

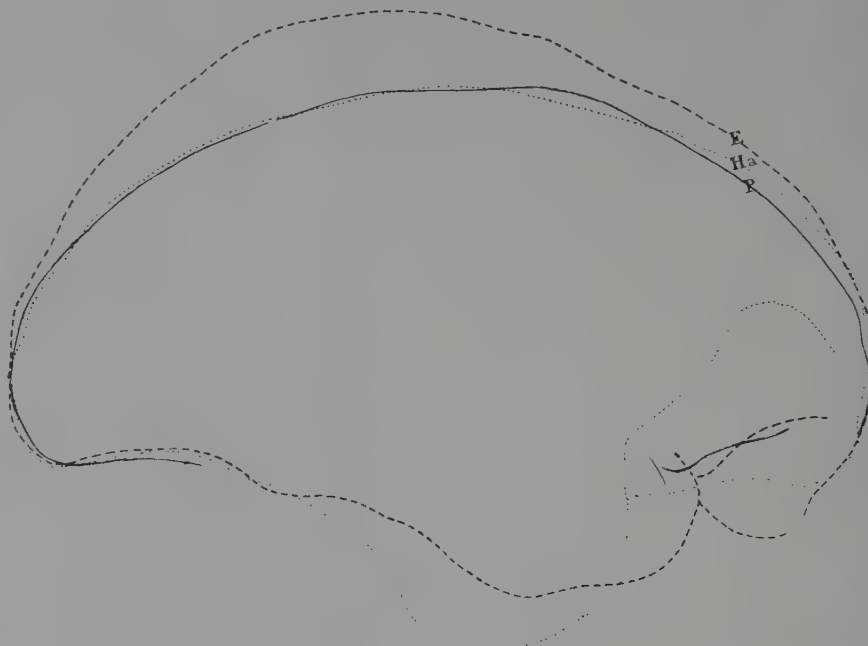
PLATE III.

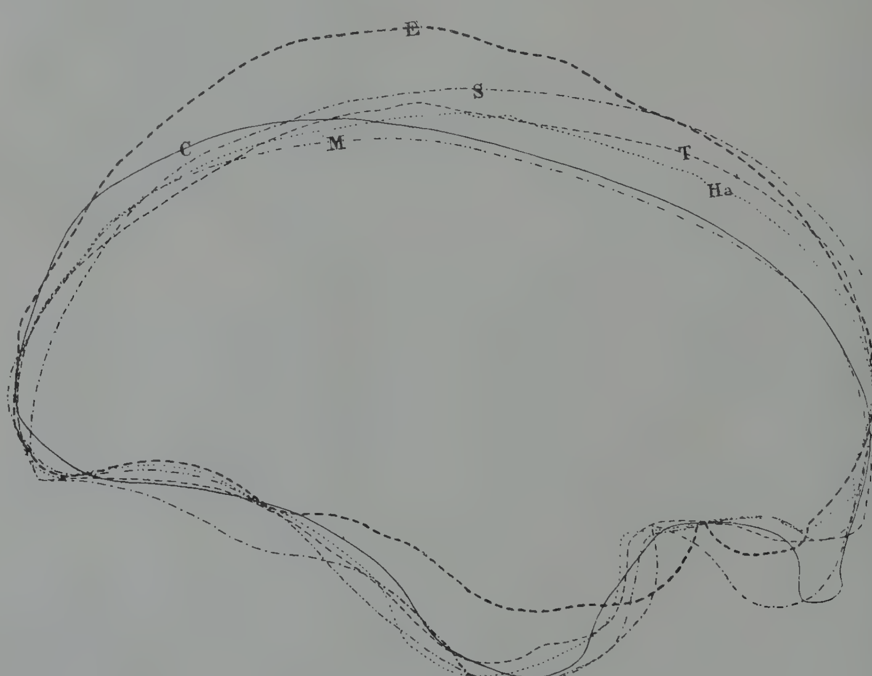


Upper Figure: Endocranial casts of adolescent *Sinanthropus* (*Si*). × 3/4, mesocephalic normal man, Dutch (*H*). Made to equal length. Inset: Microcephalic man (*M*). × 5/9.

Lower Figure: Endocranial cast of *Pithecanthropus erectus* (*P*). × 3/4, and Chimpanzee (*Ch*). Made to equal length.

Both Figures diagrammatic drawings (telephotographic outlines) of right norma lateralis views. Same orientation as in Plate II.





Upper Figure: Endocranial casts of *Pithecanthropus erectus* (*P*). $\times$ 3/4, mesocephalic Man (*E*), and *Hylobates agilis* (*Ha*). Made to equal length.
Lower Figure: Endocranial casts of mesocephalic, Dutch, Man (*E*), Chimpanzee (*T*), Orangoutan (*S*), *Hylobates agilis* (*Ha*), *Cebus* (*C*), and *Midas* (*M*). Made to equal length with *P* of the upper Figure.
Both Figures diagrammatic drawings (telephotographic outlines) of right norma lateralis views. Same orientation as in Plate II.

reduction, on account of the difference between the length of this cast and the length of the cranial cavity as measured directly by DAVIDSON BLACK. Then having verified the exactness of our copy by comparing its length, breadth and height with the corresponding measurements on the photographs and diagrams in the endocranion report, as indicated above, I calculated for the probable real volume of the original endocranion, corresponding with the cranial capacity and the volume of the brain with its envelopes, **918** c.c., by which t h i s *Sinanthropus* nearly approaches, but not really surpasses *Pithecanthropus*.

This volume approaches the lower result of the two methods of volumetric determinations applied by DAVIDSON BLACK, the one which he held for less reliable, yielding approximately 900 c.c., whereas by the other, which he considered to yield the most accurate volumetric results, he found 964 c.c.

The difference between these results being very great, and as in the present instance it indeed seemed desirable to obtain the most reliable figure, I have endeavoured to ascertain the degree of exactness of the two methods applied by the distinguished anatomist.

According to his own words (p. 264 of the report), DAVIDSON BLACK proceeded as follows:

"A number of trial restorations of the base of the endocranial cast were made and cast in plaster before the one which probably approximates most nearly to the correct form was selected. These various plaster casts were dried and waterproofed by impregnation with shellac under negative pressure. Their respective volumes were then determined in water by the displacement method in a cylindrical vessel just accommodating the specimens. As a result of repeated trials by two independent and qualified observers the volume in each approached 900 c.c., it being impossible to obtain a more accurate reading of volume displacement in a vessel of the size necessary to accomodate one cast."

"To overcome this difficulty a Negocoll cast was made of the restored endocranial cast which had been finally selected for description. This Negocoll was then melted and poured in cylindrical moulds which when set were of a size readily to be accomodated within graduated measuring cylinders. Twenty-five consecutive determinations of the volume of these Negocoll cylinders were then made in order to permit the calculation of the probable error."

In a number of trials I now found it easily possible to obtain accurate readings of volume displacement in a cylindrical vessel of 14.5 cm width, spaciously accomodating the endocranial cast, to 0.5 mm shifting of water-level, corresponding to about 8 c.c. of volume displacement, if using the right illumination and slips of white paper as indicators.

As in such vessel 64 c.c. (the difference of the results obtained by the two methods) of volume displacement correspond to about 4 mm shifting of water-level, this difference must not chiefly be imputed to the inaccuracy of this direct displacement method, as an error of observation, but obviously

for the greater part to the indirect method, that using a Negocoll endocranial cast.

Further experiments taught me that Negocoll, in the conditions of the volumetric method described by DAVIDSON BLACK, unavoidably expands by absorbing a certain quantity of water, the volume thus increasing about 5 per cent. in a short time. After this the Negocoll retains very nearly the same thus obtained volume during a much longer time than is required for twenty-five consecutive determinations of the volume.

In this way I convinced myself that the great difference between the results of the two methods was chiefly owing to the unsuitability of Negocoll for such volumetric measurements. Moreover a confirmation was thus found of the result of my own determination, as $964 \times \frac{100}{105} = 918.1$.

Such a volume of the endocranial cast or cranial capacity of 918 c.c., about equal to that of *Pithecanthropus*, is certainly a very low one for a human skull, as this *Sinanthropus* undoubtedly is. For at the age of this early adolescent human individual the volume of the brain is almost equal to that of the adult. Such a boy of 15 or 16 years of age was also the Neandertal individual of Le Moustier, whose cranial capacity is estimated to be more than 1500 c.c., but how youthful are the morphological features of his skull! The shape and the major features of this *Sinanthropus* skull, on the contrary, are those of a full grown male Neandertaler. They are undoubtedly very different from the typical skull shape and features of a child, and of a female of his race, and also from those of a pygmean race or a normal small individual of his own race. We meet here with a contradiction of cranial form and cranial capacity, a contradiction emphasized by the other *Sinanthropus* skull, attributed by DAVIDSON BLACK to an adult woman. In contradistinction with the adolescent skull it, indeed, exhibits true female features. It is difficult to estimate the capacity of this very incomplete cranium, however 1150 c.c. will probably not be too high an estimate. In proportion to such a female capacity a normal adult male of the same race should have about 1300 c.c. capacity. However the adolescent *Sinanthropus* exhibits adult morphology in combination with a brain volume very much smaller than the normal one of his age.

This is a contrast which is perfectly unconceivable if we consider this *Sinanthropus* youth as a normal individual. The conclusion appears unavoidable that the brain in this individual was not full-grown. In accordance with this the brain exhibits a feature frequently found in microcephalic brains. Clearly the Sylvian fissure had not yet attained the development of the full-grown human brain but, most conspicuously on the left hemisphere, in its inferior part still remained in the infantile condition of a fossa, the inferior frontal gyrus and the temporal lobe there being widely separated from one another, and thus the insular region left incovered by its operculae. This particular feature of this *Sinanthropus* brain is well described by DAVIDSON BLACK.

In conclusion I may express my opinion that the adolescent *Sinanthropus* is a human male, belonging to the Neandertal group of mankind, the species *Homo neandertalensis*, maybe an interesting new race, with individually imperfectly developed and hence abnormally small brain.

Further skulls will instruct us how far the striking external particularities of the Locus E skull are characteristic of a distinct race or only individual features due to unfinished ontogenesis.

Embryology. — *Über den Glykogenstoffwechsel tierischer „Organisatoren"*. Von M. W. WOERDEMAN.

(Communicated at the meeting of April 29, 1933).

In einer vorigen Mitteilung [1]) habe ich berichtet über den Glykogenstoffwechsel des Organisationszentrums in der Amphibiengastrula.

Ich fand, dasz die bei der Gastrulation invaginierten Zellen in kurzer Zeit nach der Einrollung den grössten Teil ihres Glykogens verlieren und dasz also offenbar in der Urmundlippe sehr besondere Stoffwechselverhältnisse herrschen. Ich habe die Vermutung ausgesprochen, dasz in irgend einer Weise diese Stoffwechselverhältnisse zusammenhängen könnten mit den merkwürdigen Wirkungen, die von den Urmundlippen ausgehen und welche SPEMANN veranlasst haben von einem „Organisationszentrum" in den Urmundlippen zu sprechen.

Nun giebt es verschiedene Wege um zu untersuchen, ob wirklich zwischen der Glykolyse und den Organisationswirkungen eine Beziehung besteht.

Wir haben in letzter Zeit in meinem Institute versucht die Hypothese zu prüfen, ob Organisation (Induktionswirkung) und Glykolyse in den Zellen des Organisators (Induktors) mit einander etwas zu tun haben, wobei wir tatsächlich verschiedene Wege eingeschlagen haben.

In der vorliegenden Mitteilung werde ich nur über einen dieser Wege berichten, nämlich über den histiochemischen Glykogennachweis in Zellgruppen, die als Organisatoren betrachtet werden können.

Bekanntlich hat SPEMANN die Augenblase einen sekundären Organisator genannt.

Sie soll die Linsenanlage im Ektoderm des Kopfes induzieren. Obwohl die Frage, in welcher Weise die Linseninduktion stattfindet, noch nicht vollständig geklärt ist, kann man wohl als gesichert annehmen, dasz bei den Amphibien die Augenblase auf das überlagernde Kopfektoderm eine Induktionswirkung ausübt.

Es lag nun nahe zu untersuchen, ob auch in diesem „Linsenorganisator" der Glykogenstoffwechsel besondere Verhältnisse zeigt. Ich habe deshalb mit der in der vorigen Mitteilung beschriebenen Technik (Jodreaktion nach

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[1]) Cf. diese Proceedings Vol. XXXVI, N⁰. 2, 1933.

LANGHANS) den Glykogengehalt der Augenanlage verschiedener Entwicklungsstadien vom mexikanischen Axolotl untersucht.

Auffallend ist die starke Glykogenreaktion der Medullarplatte, ihre Zellen besitzen einen grossen Gehalt an Glykogen.

Nachdem sich die Platte zum Neuralrohr geschlossen hat, findet man noch immer eine sehr deutliche Glykogenreaktion in ihren Zellen. Aber jetzt treten Unterschiede in den verschiedenen Abschnitten des Rohres auf, die so regelmässig in allen untersuchten Keimen wahrgenommen werden konnten, dasz man sie wohl verbinden muss mit gesetzmässigen chemischen Differenzierungen während der Entwicklung des Nervensystems.

Einen der erwähnten Unterschiede findet man im Gebiet der Augenanlagen. Der Boden des Vorderhirnbläschens und diejenige Abschnitte der Seitenwände, die sich zu den Augenblasen ausstülpen, besitzen deutlich mehr Glykogen als die übrigen Abschnitte der Wand. Die Jodreaktion auf Glykogen ist auch bei den älteren Stadien immer stark in der präsumptiven Regio chiasmatica und der Augenanlage.

Wenn sich aber die Augenblasen auszustülpen anfangen, da fällt es auf, dasz namentlich die ventralen Teile der Augenstiele und die dicken, dem Ektoderm zugekehrten, Wände der Augenblasen eine starke Jodreaktion zeigen, die nirgends im ganzen Prosencephalonbläschen so deutlich ist wie hier. Wohl besitzen die Zellen der medialen Wand der primären Augenblase noch sehr viel Glykogen, aber sie verlieren einen Teil bald darauf. Konstant findet man das Glykogen angehäuft in denjenigen Abschnitten der Zellen, die dem Lumen des Augenbläschens zugekehrt sind. Dieser Lokalisation will ich keine Bedeutung beimessen, denn sie kann, wie ich schon in der vorigen Mitteilung beschrieb, künstlich entstanden sein. Aber sie verursacht in den mikroskopischen Schnitten ein sehr auffallendes Bild.

Das zeigt die schematische Abbildung 1, worin das Glykogen, so wie es die Jodreaktion aufdeckt, durch Punktierung angegeben ist. In diesem Stadium ist in der lateralen Wand der Augenblase und ventral im Augenstiel der Glykogengehalt am grössten.

Bei der Bildung des Augenbechers aus der Vesicula optica verschwindet das Glykogen zum Teil aus der Zellschicht des Tapetums.

Die Regio chiasmatica bleibt kenntlich durch ihre deutliche Glykogenreaktion, ebenso wie die Abschnitte des Retinablattes, die sich dem Tapetum angeschmiegt haben, und die ventrale Wand des Augenstieles.

Wenn nun eine Verdickung des Kopfektoderms (das ziemlich viel Glykogen in seinen Zellen enthält) die Anlage der Augenlinse anzeigt, so ist in der Retina eine sehr auffallende Erscheinung aufgetreten. Ihr Zentrum verliert sein Glykogen und in den Präparaten sieht dieser Abschnitt deutlich hell aus. Dagegen bleibt die starke Glykogenreaktion erhalten in den Umschlagrändern des Augenbechers, also in den präsumptiven Irisrändern. Dabei ist sie im oberen Rande viel deutlicher als unten, denn im Gebiet der fötalen Augenspalte ist der Glykogenschwund auch sehr deutlich wahrzunehmen.

Die Linsenwucherung des Ektoderms, die reichlich Glykogen enthält, kontrastiert nun in bemerkenswerter Weise mit dem fast glykogenfreien Retinazentrum, womit sie in Berührung tritt.

Abb. 1.

a. Schnitt durch die Augenanlage eines jungen Axolotlkeimes. (Glykogenreaktion nach LANGHANS).

b. Neuralrohr (vorderer Abschnitt) eines ähnlichen Keimes mit der für Schnitt a. verwandten Schnittrichtung.

Ich gehe auf die weiteren Veränderungen des Glykogengehaltes nicht ein. Es genügt für meinen Zweck gezeigt zu haben, dasz auch in der Augenblase bei ihrer Entwicklung merkwürdige Unterschiede im Glykogengehalt gefunden werden. Einerseits ist bemerkenswert der hohe Gehalt an Glykogen der Augenanlagen und der zwischen ihnen liegenden Regio chiasmatica. Anderseits muss auch in diesem Fall die Aufmerksamkeit gelenkt werden auf einen Glykogenschwund, der zeitlich zusammenfällt mit dem Sichtbarwerden einer Linsenwucherung. Diese Glykolyse tritt auf im Zentrum der Retina-Anlage und in der Umgebung der Augenspalte.

Eine starke Glykogenreaktion bleibt vorhanden in den Irisrändern, namentlich im Oberrande.

Zweifelsohne ist die Glykolyse nur ein Teilprozess der verwickelten chemischen Prozesse, die sich in den embryonalen Zellen vollziehen. Sie ist aber durch unsre histiochemischen Methoden leicht nachweisbar, während andere Prozesse schwieriger entdeckt werden können.

Ich will denn auch in der Glykolyse nicht (wenigstens vorläufig nicht) den essentiellen Prozess sehen, der bei der Gastrulation in der oberen Urmundlippe und bei der Linsenbildung in dem Retinablatte des Augenbechers stattfindet, sondern nur den Ausdruck besonderer Stoffwechselvorgänge, wovon die Glykolyse nur einen Teilprozess darstellt.

Dann kann ich also aus meinen Wahrnehmungen den Schluss ziehen, dasz sowohl in dem Organisationszentrum in der Urmundlippe, wie in dem

„Linsenorganisator" typische Stoffwechselvorgänge, die mit Glykolyse einhergehen, auftreten.

Das regt uns zu weiteren Untersuchungen an, wobei geprüft werden soll, ob diese Prozesse mit der Induktionswirkung in Beziehung stehen, und was dabei die Bedeutung der Glykolyse ist.

Über einige, zum Teil schon ausgeführten, Experimentenreihen hoffe ich demnächst zu berichten.

Anatomisch-Embryologisches Institut der Universität Amsterdam.

Physics. — *A direct method for the measurement of low air speeds.* By M. ZIEGLER. (Mededeeling N^0. 28 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft. (Communicated by Prof. J. M. BURGERS.)

(Communicated at the meeting of April 29, 1933).

For the investigation of the motion of fluids frequently use is made of foreign elements which are carried along by the flow, their displacement being observed either visually or by means of some special apparatus [1]). A necessary condition for this kind of work is that the convection velocity of these foreign elements must be equal to the fluid velocity itself. In the case of a stationary rectilinear motion this condition will be fulfilled provided that no other forces are working on the convected objects than the forces originating from the surrounding medium, whereas in the case of accelerations of the fluid, the particles moreover must be of the same density as the fluid.

In the following lines a method for the measurement of low air velocities according to this principle will be described, which consists in the heating of a small portion of the moving air during a very short time, and the detection of the hot air, a certain time afterwards, at some distance downstream. The method is suitable for the determination of the velocity of a steady and horizontal airflow, provided that the velocity does not change in the direction of motion.

The condition that the velocity of heat convection in the direction of flow is equal to the velocity of the air certainly is satisfied here. It cannot be denied, however, that heat conduction and free convection have some disturbing influence to which we will return later. These effects limit the range of velocities which can be measured with sufficient exactness, as the

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1) We must confine ourselves here to mention only the work of H. C. H. TOWNEND, whose methods, like the one described in the present paper, are based on the heating of small portions of air. See: Aeron. Res. Comm. Rep. & Mem. Nos 1349 and 1434: On rendering airflow visible by means of hot wires; Hot Wire and Spark Shadowgraphs of the Airflow through an Airscrew.

imposed convection, which we intend to measure, must be great compared to the fading, the deformation and the displacements due to conduction and to free convection. The air velocity consequently may not be too low; experimentally it appeared that the lowest velocities which could be measured were of the order of 10 cm/sec. As to the higher velocities the limit is determined by the steadiness of the airstream as obtained under the circumstance of the experiment.

The realisation of the method was very simple. As source of heat (emitter) a grid is used which consists of 7 parallel platinum-iridium wires of 60 mm length and 0.050 mm diameter, mounted in a plane perpendicular to the direction of the stream, at distances of ca 3 mm from each other. The disturbing effect of the grid itself may be neglected as the distance between the wires is equal to 60 times the wire diameter and the resistance brought into the flow thus is very small. This supposition was verified experimentally for high velocities by means of a PITOT tube. It is evident that the frame of the grid must be so constructed that it practically offers no resistance to the air [1]). The grid can be heated for a short interval of time by discharging through it a condensor of a few μF charged up to a potential of about 200 V [2]). For the detection of the hot air after it has moved a certain distance downstream, a platinum wire of 0.005 mm diameter and 20 mm length is used, through which flows a current of a few mA. This current is too low for appreciably heating the wire, so that its sensitivity as an anemometer is very small; any variation of the temperature of the air, however, causes a proportional variation of the electrical resistance of the wire and thus influences the magnitude of the current or of the voltage drop.

With a suitable rapid oscillograph the velocity of the air now can be determined simply by recording both the discharge of the condensor and the decrease of the current through the wire; the distance between emitter and receiver divided by the time difference then gives the velocity of the air.

The use of an oscillograph and the necessity of photographic recording would make this method rather cumbersome, and would limit its application to laboratory work. In order to arrive at a practical method of measurement it must be possible to read off the value to be measured after a few manipulations; the arrangement, besides, has to be kept as simple, compact

---

[1]) Measurements also were made with smaller grids, consisting of wires of 30 mm length. The thin bars of the frame, carrying the insulated strips between which the wires are stretched, then were at a distance of ca 30 mm from each other, and with a receiving wire of 20 mm length downstream of the grid, it appeared that the air velocity measured was somewhat too low, which must be ascribed to the influence of the wake of the frame. For this reason the frame was made larger, so that the air which passes the receiving wire, now is no longer disturbed by it.

[2]) A wire can be brought instantaneously at a high temperature, independently of its heat capacity, by suitably selecting the time constant of the electrical system formed by condensor and wire.

and cheap as possible. For this reason an apparatus has been made, which has these advantages and works satisfactorily.

In this apparatus the measurement of time-intervals has been replaced by the determination of the coincidence of the passage of the heated cloud across the receiving wire, with a "signal", emitted a definite interval of time after the discharge of the condensor. If the discharge is repeated regularly, say n times per second, the discharges themselves may be used as signals. If now the passage of each cloud of hot air coincides with the discharge following that one which produced the cloud, the time of convection for the distance l between grid and receiver is equal to l/n sec. The velocity of the air then is equal to $n \times l$.

For the detection of the coincidences, the arrangement of fig. 1 is used. Its application is not bound to the problem described here; it may be used in many cases where coincidences of two phenomena must be detected.

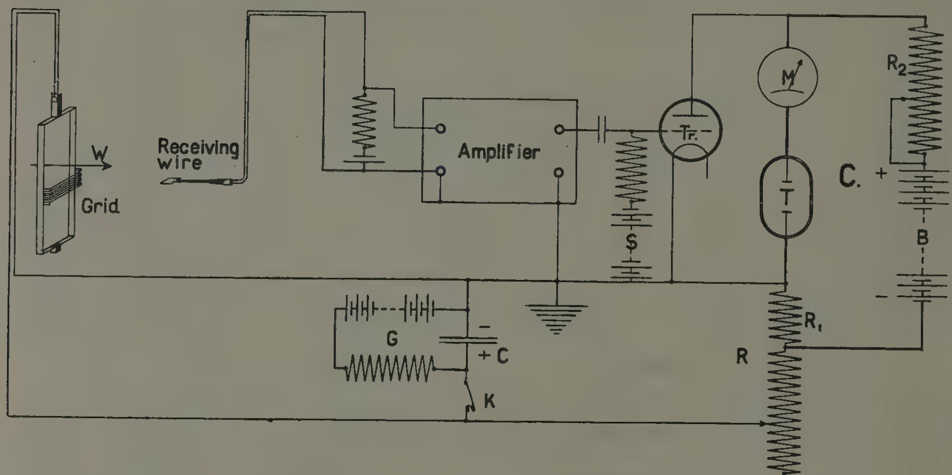

Fig. 1. Experimental arrangement for the measurement of low airspeeds.

T Gasfilled tube (neon tube); ignition potential $= 122\ V$, maintaining potential $= 100\ V$.
M Milliammeter for 2 mA.
R_2 Variable resistance of 25000 Ω; $R_1 = 500\ \Omega$.
B Battery of ca 115 V. S: C-battery of ca 20 V.
Tr Philips tube A 415.
C Condensor of 6 μF. The circuit G of condensor C, battery and resistance was a Philips plate supply N^0. 4003.
K Arrangement for periodically discharging the condensor C.

In the circuit C. a gasfilled tube T and a milliammeter M are inserted. The tension E of the battery B is below the ignition potential V_i of the tube but above the potential V_m necessary for maintaining the glow discharge; thus when ignition once has been started the discharge through the tube continues until it is interrupted by a special cause. The current through the milliammeter M then has the value $i = \frac{E - V_m}{R_1 + R_2}$ provided that the

resistance R_{tr} between kathode and anode of the triode Tr is infinite; this is ensured by a strongly negative value of the grid potential of the triode. When R_{tr} gets a lower value the current through the tube decreases; as soon as it reaches zero the glowing is extinguished, and R_{tr} may become infinite again without a new ignition taking place, as the battery potential E is too low for it. If we now push the key K, the condensor C (which is charged up to a high potential) suddenly discharges through the grid and partially also through the resistance R, the part R_1 of which is inserted in the circuit of the tube. The potential between the electrodes of the tube now for a very short time rises above the ignition potential and the tube starts glowing, while at the same instant the major part of the electrical energy of the condensor is in the grid transformed into heat, which is carried off by the air. As soon as the hot air reaches the receiver the increase of temperature of the wire, via a simple classical resistance coupled amplifier, causes a considerable increase of the grid potential of the triode Tr and a current flows through it for a short time. The amplification must be such that the maximum intensity of this current is greater than i so that the tube stops glowing. The time of glowing then has been equal to the time τ between emission and reception and thus is proportional to the distance l between emitter and receiver. For a position of the wire very near to the grid this time is very short and the milliammeter only gets a small current impulse theoretically equal to $i\tau = i\frac{l}{W}$, W being the air speed. Now the condensor is discharged n times per second and thus a slow meter will indicate an average current $I = n\,i\frac{l}{W}$. This average current increases proportionally with l. For $l = \frac{W}{n}(1-\varepsilon)$, where $\varepsilon \ll 1$, we get $I \cong i$, because then the tube ignites again a very short time after it has stopped glowing. For $l = \frac{W}{n}(1+\varepsilon)$ the state of things suddenly is reversed: the current now each time becomes zero immediately after the ignition of the tube and $I \cong 0$. In fig. 2 the various possible cases are illustrated schematically.

We thus see that when the distance between emitter and receiver is increased with regular steps, the indication of the milliammeter rises until it has reached a value $\cong i$, and then suddenly falls off to zero. The sudden fall of the current is a criterion for the position of the receiving wire at a distance W/n from the grid and also for distances equal to 2, 3...... times W/n.

This apparatus thus allows a direct measurement of the velocity; observing the milliammeter one has only to find out the critical position (or positions) of the receiver. The velocity of the air then is equal to n times the first critical distance.

For n the value 4 has been chosen: the period must be great compared with the lag of the tube etc., and still so short that the distance W/n is

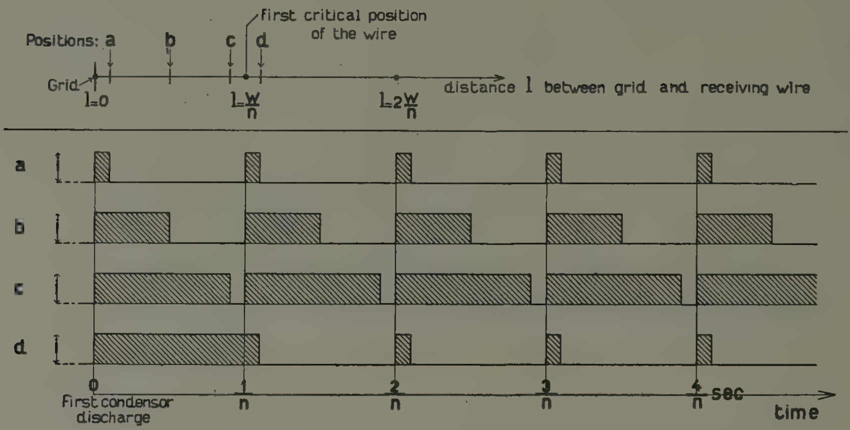

Fig. 2. Typical example of the consecutive current impulses through the milliammeter for 4 positions of the receiving wire.

not too great. Though the successive current impulses are not averaged entirely by the milliammeter used, fluctuations of the frequency n per second being still visible, the falling off of the current at the critical position can be observed easily.

For the first experiments a grid and receiving wire were placed in a 4 inch tube through which air could be sucked by means of a small fan. Much care was taken to obtain a flow as regular as possible, but the velocity itself then could not be determined independently. In order to check the apparatus some measurements therefore were made by moving the grid and the receiver with a known velocity through air at rest. For these experiments the empty water tank of the laboratory (length 8 m; section upper side, a narrow slit being left open. A carriage rides on rails above the $1 \times 0.8\ m^2$) was used. The tank was covered with cardboard panels at the tank, and emitter and receiver were fixed to it by means of bars which passed through the slit, the distance above the bottom of the tank being about 30 cm. The carriage can move with various constant velocities.

Fig. 3 gives experimental results for a velocity of about 30 cm/sec. As regards the sharpness of the critical position the results are somewhat disappointing. This can be caused by all kinds of irregularities: motion of the air in the tank and inequalities of its temperature, variations in the motion of the carriage, while also vibrations of the grid and of the wire are fairly important and produce effects which cannot be overlooked. It must be remarked besides that, just in the critical position, the apparatus cannot average and all fluctuations come through. From a simple graph like fig. 3 the velocity of the air, however, can be deduced exactly.

As stated before, diffusion and free convection have some disturbing effects. The free convection reveals itself as an upward motion of the hot

air, which is not to be neglected: the receiving wire therefore must be raised slightly as the distance l increases. In order to reduce the effect of

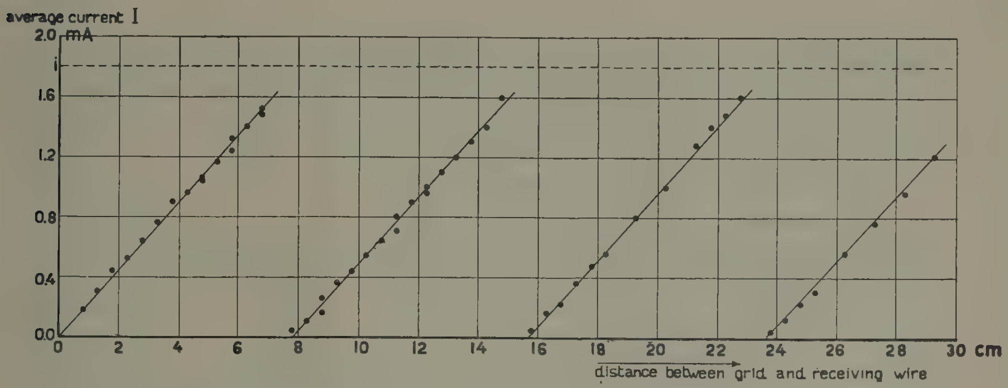

Fig. 3. Observed values of the average current I through the mA meter as a function of the distance between grid and wire for a relative air velocity of 31,2 cm/sec and $n = 4$ $\left(\text{first critical distance} = \frac{31{,}2}{4} = 7{,}8 \text{ mm}\right)$. $i = 1{,}8$ mA.

free convection, the temperature of the emission is kept as low as possible, and thus for a given charging potential, the condensor C is taken so small as the sensitivity and steadiness of the arrangement will allow. In the present arrangement the upward motion appeared to be of the order of 25 mm/sec.

We have restricted ourselves to the question whether the proposed method could be used practically for the measurement of low velocities of air. It must be remarked that during the first experiments, which were executed with an oscillograph, it could be observed that the cloud of hot air deforms very much. When the emitter was heated very strongly, sometimes two successive maxima of temperature could be observed at a certain distance downstream from the emitter, and also the presence of an accelerated quantity of air coming after the hot air (this was detected by the cooling of the receiving wire, the electric current through which then had a greater intensity, so that the wire acted as an anemometer). At the upper boundary of the cloud only one small region of increased temperature is observed in all cases: the receiver therefore preferably must be placed as high as possible.

A more detailed investigation of the behaviour of a quantity of air of higher temperature carried along by an horizontal airflow apparently may yield some interesting results.

Mathematics. — *On the Rational Solution of the matrix Equation $SX=XT$.* By Dr. D. E. RUTHERFORD. (Communicated by Prof. R. WEITZENBÖCK.)

(Communicated at the meeting of March 25, 1933).

1. In a recent paper [1]), I gave a rational solution to the matrix equation

$$AX=XA \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A\,1)$$

and it was shown that the solution depends upon solving equations of the type

$$B_n\, Y_{n,m}=Y_{n,m}\, B_m\,, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A\,4)$$

where

$$B_n=\begin{bmatrix} . & 1 & . & & . \\ . & . & 1 & & . \\ & & \cdot & & \\ . & . & . & & 1 \\ \alpha_0 & \alpha_1 & \alpha_2 & \ldots & \alpha_{n-1} \end{bmatrix} \text{ and } B_m=\begin{bmatrix} . & 1 & . & & . \\ . & . & 1 & & . \\ & & & & \\ . & . & . & & 1 \\ \beta_0 & \beta_1 & \beta_2 & \ldots & \beta_{m-1} \end{bmatrix}$$

are simple rational canonical matrices of orders n and m respectively and where the unknown matrix $Y_{n,m}=[y_{ij}]$ has n rows and m columns. It was further shown that the solution to $(A4)$ can always be given in the form

$$Y_{n,m}=I_{n,m}\,(y_{11}\,I+y_{12}\,B+\ldots.+y_{1,m}\,B^{m-1});. \quad . \quad . \quad . \quad (A\,11)$$

if, however, $n<m$, the elements $y_{11}, y_{12}, \ldots, y_{1m}$ are not independent. In the same way the solution of the more general equation

$$SX=XT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B\,1)$$

where S and T are distinct square matrices not necessarily of the same order, again devolves upon the solution of equations of the type (A 4); and the solution can again be given in the form of $(A\,11)$, but in this case also, the elements $y_{11}, y_{12}, \ldots, y_{1m}$ are not necessarily independent. A large part of this paper will therefore be concerned with discussing

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[1]) Proceedings of the Koninklijke Akademie van Wetenschappen Vol. XXV (1932), p. 870. Formulae from this paper will be denoted by the prefix A, thus $(A4)$ means equation (4) in the above paper.

See also O. SCHREIER and B. L. v. D. WAERDEN, Abhand. Math. Seminar, Hamburg Vol. 6 (1928) p. 308.

he interdependence of the elements $y_{11}, y_{12}, \ldots, y_{1m}$. This depends upon he rank of the matrix of the following equations

$$\left.\begin{aligned} y_{i,1} &= \beta_0 \quad y_{i-1,m} \\ y_{i,j} &= \beta_{j-1}\, y_{i-1,m} + y_{i-1,j-1} \end{aligned}\right\} \qquad (A6)$$

$$\left.\begin{aligned} \alpha_0\, y_{1,1} + \ldots\ldots + \alpha_{n-1}\, y_{n,1} &= \beta_0 \quad y_{n,m} \\ \alpha_0\, y_{1,j} + \ldots\ldots + \alpha_{n-1}\, y_{n,j} &= \beta_{j-1}\, y_{n,m} + y_{n,j-1} \end{aligned}\right\} \qquad (A7)$$

where $j \neq 1$ and $i \neq n$.

When solving the equations $(A\,1)$, it was found that certain relations $(A\,8)$ between the α's and the β's exist; but these relations no longer hold when we solve the more general equation $(B\,1)$. It is found convenient to simplify matters by means of a new notation which will be explained in paragraph 3.

Throughout this paper I_n denotes the unit matrix of order n and U_n denotes the auxiliary unit matrix of order n, thus-

$$I_4 = \begin{bmatrix} 1 & . & . & . \\ . & 1 & . & . \\ . & . & 1 & . \\ . & . & . & 1 \end{bmatrix} \text{ and } U_4 = \begin{bmatrix} . & 1 & . & . \\ . & . & 1 & . \\ . & . & . & 1 \\ . & . & . & . \end{bmatrix}.$$

As in the previous paper we find it convenient to write $-1 = \alpha_n = \beta_m$.

2. As a simple case we shall discuss the rank of $Y_{n.m}$ when $m = 5$ and $n = 3$. Suppose then, that

$$B_3 = \begin{bmatrix} . & 1 & . \\ . & . & 1 \\ \alpha_0 & \alpha_1 & \alpha_2 \end{bmatrix} \text{ and } B_5 = \begin{bmatrix} . & 1 & . & . & . \\ . & . & 1 & . & . \\ . & . & . & 1 & . \\ . & . & . & . & 1 \\ \beta_0 & \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{bmatrix}$$

and that the irrational canonical forms of B_3 and B_5 are

$$C_3 = L_3^{-1}\, B_3\, L_3 = \left[\begin{array}{cc:c} \nu_1 & 1 & . \\ . & \nu_1 & . \\ \hdashline . & . & \nu_2 \end{array}\right] \qquad (B\,2)$$

$$C_5 = L_5^{-1}\, B_5\, L_5 = K_5^{-1}\, B'_5\, K_5 = \left[\begin{array}{ccc:cc} \mu_1 & 1 & . & . & . \\ . & \mu_1 & 1 & . & . \\ . & . & \mu_1 & . & . \\ \hdashline . & . & . & \mu_2 & 1 \\ . & . & . & . & \mu_2 \end{array}\right] \qquad (B\,3)$$

where L_3, L_5 and K_5 are all nonsingular matrices. The matrix $E_{3,5}$ of equations $(A\,6)$ and $(A\,7)$ can then be written

$$
\left[\begin{array}{ccc|ccc|ccc|ccc|ccc}
\cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_0 & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_0 & \cdot \\
\alpha_0 & \alpha_1 & \alpha_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_0 \\
\hline
-1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_1 & \cdot & \cdot \\
\cdot & -1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_1 & \cdot \\
\cdot & \cdot & -1 & \alpha_0 & \alpha_1 & \alpha_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_1 \\
\hline
\cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & -\beta_2 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & -\beta_2 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -1 & \alpha_0 & \alpha_1 & \alpha_2 & \cdot & \cdot & \cdot & \cdot & \cdot & -\beta_2 \\
\hline
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & 1 & \cdot & -\beta_3 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & 1 & \cdot & -\beta_3 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \alpha_0 & \alpha_1 & \alpha_2 & \cdot & \cdot & -\beta_3 \\
\hline
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & -\beta_4 & 1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot & -\beta_4 & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \alpha_0 & \alpha_1 & \alpha_2-\beta_4
\end{array}\right]
\quad \text{or} \quad
\left[\begin{array}{c|c|c|c|c}
B_3 & \cdot & \cdot & \cdot & -\beta_0 I_3 \\
\hline
-I_3 & B_3 & \cdot & \cdot & -\beta_1 I_3 \\
\hline
\cdot & -I_3 & B_3 & \cdot & -\beta_2 I_3 \\
\hline
\cdot & \cdot & -I_3 & B_3 & -\beta_3 I_3 \\
\hline
\cdot & \cdot & \cdot & -I_3 & B_3-\beta_4 I_3
\end{array}\right]
$$

where we partition $E_{3,5}$ into submatrices as indicated by the thin lines. Now, by $(B\,2)$

$$
\begin{bmatrix}
L_3^{-1} & \cdot & \cdot & \cdot & \cdot \\
\cdot & L_3^{-1} & \cdot & \cdot & \cdot \\
\cdot & \cdot & L_3^{-1} & \cdot & \cdot \\
\cdot & \cdot & \cdot & L_3^{-1} & \cdot \\
\cdot & \cdot & \cdot & \cdot & L_3^{-1}
\end{bmatrix}
\begin{bmatrix}
B_3 & \cdot & \cdot & \cdot & -\beta_0 I_3 \\
-I_3 & B_3 & \cdot & \cdot & -\beta_1 I_3 \\
\cdot & -I_3 & B_3 & \cdot & -\beta_2 I_3 \\
\cdot & \cdot & -I_3 & B_3 & -\beta_3 I_3 \\
\cdot & \cdot & \cdot & -I_3 & B_3-\beta_4 I_3
\end{bmatrix}
\begin{bmatrix}
L_3 & \cdot & \cdot & \cdot & \cdot \\
\cdot & L_3 & \cdot & \cdot & \cdot \\
\cdot & \cdot & L_3 & \cdot & \cdot \\
\cdot & \cdot & \cdot & L_3 & \cdot \\
\cdot & \cdot & \cdot & \cdot & L_3
\end{bmatrix}
=
\begin{bmatrix}
C_3 & \cdot & \cdot & \cdot & -\beta_0 I_3 \\
-I_3 & C_3 & \cdot & \cdot & -\beta_1 I_3 \\
\cdot & -I_3 & C_3 & \cdot & -\beta_2 I_3 \\
\cdot & \cdot & -I_3 & C_3 & -\beta_3 I_3 \\
\cdot & \cdot & \cdot & -I_3 & C_3-\beta_4 I_3
\end{bmatrix}.
$$

Call this last matrix $F_{3,5}$ and let $G_{3,5}$ be the matrix obtained from $F_{3,5}$ by taking for the rows and columns of $G_{3,5}$ in succession the 1st, 4th, 7th, 10th, 13th, 2nd, 5th, 8th, 11th, 14th, 3rd, 6th, 9th, 12th, 15th, rows and columns of $F_{3,5}$. It is easy to show that

$$
G_{3,5} = \begin{bmatrix}
\nu_1 I_5 - B'_5 & I_5 & \cdot \\
\cdot & \nu_1 I_5 - B'_5 & \cdot \\
\cdot & \cdot & \nu_2 I_5 - B'_5
\end{bmatrix},
$$

and that

$$
\begin{bmatrix}
K_5^{-1} & & \\
& K_5^{-1} & \\
& & K_5^{-1}
\end{bmatrix}
\begin{bmatrix}
\nu_1 I_5 - B'_5 & I_5 & \cdot \\
\cdot & \nu_1 I_5 - B'_5 & \cdot \\
\cdot & \cdot & \nu_2 I_5 - B'_5
\end{bmatrix}
\begin{bmatrix}
K_5 & \cdot & \cdot \\
\cdot & K_5 & \cdot \\
\cdot & \cdot & K_5
\end{bmatrix}
=
$$

$$
= \begin{bmatrix}
\nu_1 I_5 - C_5 & I_5 & \cdot \\
\cdot & \nu_1 I_5 - C_5 & \cdot \\
\cdot & \cdot & \nu_2 I_5 - C_5
\end{bmatrix}
$$

Let us call this matrix $H_{3,5}$. The ranks of $E_{3,5}$, $F_{3,5}$, $G_{3,5}$ and $H_{3,5}$ are the same. It is easy to see that the rank of $\nu_i I_5 - C_5$ is 5 if $\nu_i \neq \mu_1$, $\nu_i \neq \mu_2$ and is 4 if $\nu_i = \mu_1$ or μ_2. It is now easy to find the rank of $H_{3,5}$ and hence that of $E_{3,5}$.

3. We now pass on to the general case and introduce the new notation. Suppose that R is a matrix of n rows and m columns whose i, j th element is ϱ_{ij} and that P is another matrix of r rows and s columns. Now if the matrix Q of nr rows and ms columns can be partitioned into submatrices each of r rows and s columns such that the i, j th submatrix is of the form $\varrho_{ij}\, P$ then we shall write[1])

$$Q = R \langle P \rangle = \langle P \rangle R.$$

In general, however $R \langle P \rangle \neq \langle R \rangle P$.

Evidently the matrix $E_{3,5}$ of section (2) may be written

$$E_{3,5} = I_5 \langle B_3 \rangle - B'_5 \langle I_3 \rangle.$$

Similarly in the general case the matrix of the equations ($A6$) and ($A7$) can be written

$$E_{n,m} = I_m \langle B_n \rangle - B'_m \langle I_n \rangle.$$

Let C_m and C_n be the classical canonical forms of B_m and B_n respectively, where

$$C_m = L_m^{-1}\, B_m\, L_m = K_m^{-1}\, B'_m\, K_m \text{ and } C_n = L_n^{-1}\, B_n\, L_n\,.$$

Then, let

$$F_{n,m} = (I_m \langle L_n^{-1} \rangle)(I_m \langle B_n \rangle - B'_m \langle I_n \rangle)(I_m \langle L_n \rangle) = I_m \langle C_n \rangle - B'_m \langle I_n \rangle$$

and let $G_{n,m}$ be the new matrix[2]) obtained from $F_{n,m}$ by taking for the rows and columns of $G_{n,m}$ in succession the 1st, $(n+1)$th, $(2n+1)$th $\ldots, ((m-1)\,n+1)$th, 2nd, $(n+2)$th, $\ldots, ((m-1)\,n+2)$th, $\ldots$, nth, 2nth, $\ldots$, mn th rows and columns of $F_{n,m}$. Then the rank of $F_{n,m}$ is the same as that of $G_{n,m}$ where

$$G_{n,m} = \langle I_m \rangle C_n - \langle B'_m \rangle I_n\,.$$

Now, if

$$H_{n,m} = (I_n \langle K_m^{-1} \rangle)(C_n \langle I_m \rangle - \langle B'_m \rangle I_n)(I_n \langle K_m \rangle) = C_n \langle I_m \rangle - I_n \langle C_m \rangle$$

then the matrices $E_{n,m}$, $F_{n,m}$, $G_{n,m}$ and $H_{n,m}$ have all the same rank.

Suppose that all the distinct latent roots of C_n and C_m are the following

$$\lambda_1, \lambda_2, \ldots, \lambda_t,$$

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1) C.f. L. E. DICKSON. Algebras and their Arithmetics, p. 119.

2) WILLIAMSON, Bulletin Amer. Math. Soc. 37 (1931) p. 586.

and that the latent root λ_i occurs n_i times in C_n and m_i times in C_m. Also, let k_i be the lesser of n_i and m_i.

It is obvious that $\Sigma n_i = n$ and $\Sigma m_i = m$. Let $\Sigma k_i = k$. If λ_i is not one of the latent roots of C_n, then $n_i = k_i = 0$, while if λ_i is not one of the latent roots of C_m, then $m_i = k_i = 0$. A little consideration will show that the rank of the matrix $H_{n,m}$ is equal to the sum of the ranks of its submatrices $H_{n_i,m}$ where

$$H_{n_i,m} = (\lambda_i I_{n_i} + U_{n_i}) \langle I_m \rangle - I_{n_i} \langle C_m \rangle$$

and by a further rearrangement of rows and columns whithin each of the above submatrices in the same manner as before we can show that the rank of $H_{n_i,m}$ is the same as the rank of

$$I_m \langle \lambda_i I_{n_i} + U_{n_i} \rangle - C_m \langle I_{n_i} \rangle ,$$

the rank of which is the sum of the ranks of its submatrices

$$\begin{aligned} H_{n_i m_j} &= I_{m_j} \langle \lambda_i I_{n_i} + U_{n_i} \rangle - (\lambda_j I_{m_j} + U_{m_j}) \langle I_{n_i} \rangle \\ &= I_{m_j n_i} (\lambda_i - \lambda_j) + I_{m_j} \langle U_{n_i} \rangle - U_{m_j} \langle I_{n_i} \rangle . \end{aligned}$$

The rank of H_{n_i,m_j} is, in general, $m_j n_i - k_i \delta_{ij}$. For, if $i \neq j$, then $\lambda_i \neq \lambda_j$ and the rank of the matrix is $m_j n_i$. If, on the other hand, $i = j$, then the first term vanishes and the rank of matrix H_{n_i,m_j} is

$$\text{maximum } \{ m_i (n_i - 1), n_i (m_i - 1) \} = m_i n_i - k_i .$$

Hence the rank of $H_{n,m} = \sum_{ij} m_j n_i - k_i \delta_{ij} = \sum_j m_j \sum_i n_i - \sum_i k_i = m n - k$. It follows that $m n - k$ is the rank of $E_{n,m}$ and hence there are k of the elements of $E_{n,m}$ which are independent. But we have already shown in the previous paper that every element of $E_{n,m}$ is dependent upon the m elements $y_{11}, \ldots, y_{1m}$. Hence, there are $m - k$ relations existing between the m elements $y_{11}, \ldots, y_{1m}$ and in the next section we shall find these in a simple form.

(4). Let $M(\lambda) = \prod_{i=1}^{t} (\lambda - \lambda_i)^{m_i}$ and $N(\lambda) = \prod_{i=1}^{t} (\lambda - \lambda_i)^{n_i}$ be the characteristic functions of B_m and B_n respectively. Then cancelling out $\prod_{i=1}^{t} (\lambda - \lambda_i)^{k_i}$ we obtain

$$\frac{M(\lambda)}{N(\lambda)} = \prod_{i=1}^{t} \frac{(\lambda - \lambda_i)^{m_i - k_i}}{(\lambda - \lambda_i)^{n_i - k_i}}$$

where for any i, either $m_i - k_i$ or $n_i - k_i$ is zero. The degree of the

numerator $\prod_{i=1}^{t}(\lambda - \lambda_i)^{m_i - k_i}$ is $m - k$. We shall now show that corresponding to every root of

$$D(\lambda) = \prod_{i=1}^{t}(\lambda - \lambda_i)^{m_i - k_i} = 0$$

there exists a simple relation between the elements

$$y_{11}, \ldots \ldots, y_{1m}.$$

Suppose then that λ_i is an m_i-fold repeated root of $M(\lambda) = 0$ and is a k_i-fold repeated root of $N(\lambda) = 0$.

It follows that the first $m_i - 1$ derivatives of $M(\lambda)$ and the first $k_i - 1$ derivatives of $N(\lambda)$ all vanish but that the m_ith derivative of $M(\lambda)$ and the k_ith derivative of $N(\lambda)$ do not vanish.

Let us adopt the notation

$$O_\nu(\lambda) = y_{\nu 1} + \lambda y_{\nu 2} + \lambda^2 y_{\nu 3} + \ldots + \lambda^{m-1} y_{\nu m}$$

By equation $(A6)$

$$\begin{aligned} O_\nu(\lambda) = \beta_0 y_{\nu-1,m} + (y_{\nu-1,1} + \beta_1 y_{\nu-1,m})\lambda + \ldots + \\ + (y_{\nu-1,m-1} + \beta_{m-1} y_{\nu-1,m})\lambda^{m-1} \\ = \lambda O_{\nu-1}(\lambda) + y_{\nu-1,m} M(\lambda). \end{aligned}$$

By a repetition of this process

$$\begin{aligned} O_\nu(\lambda) &= \lambda^2 O_{\nu-2}(\lambda) + (y_{\nu-1,m} + \lambda y_{\nu-2,m}) M(\lambda) \\ &= \ldots \ldots \ldots \ldots \ldots \\ &= \lambda^{\nu-1} O_1(\lambda) + (y_{\nu-1,m} + \ldots + y_{1,m}\lambda^{\nu-2}) M(\lambda). \end{aligned}$$

Now since λ_i is an m_i-fold repeated root of $M(\lambda) = 0$ it follows that λ_i is also an m_i-fold repeated root of the function

$$O_\nu(\lambda) - \lambda^{\nu-1} O_1(\lambda)$$

and hence

$$\left.\begin{aligned} O_\nu(\lambda_i) &= \lambda_i^{\nu-1} O_1(\lambda_i) \\ \left[\frac{\partial}{\partial\lambda}(O_\nu(\lambda))\right]_{\lambda=\lambda_i} &= \left[\frac{\partial}{\partial\lambda}(\lambda^{\nu-1} O_1(\lambda))\right]_{\lambda=\lambda_i} \\ \ldots \ldots & \ldots \ldots \\ \left[\frac{\partial^{m_i-1}}{\partial\lambda^{m_i-1}}(O_\nu(\lambda))\right]_{\lambda=\lambda_i} &= \left[\frac{\partial^{m_i-1}}{\partial\lambda^{m_i-1}}(\lambda^{\nu-1} O_1(\lambda))\right]_{\lambda=\lambda_i} \end{aligned}\right\} \quad . \quad (B4)$$

5. By writing $y_{no} = 0$, we can write each of the equations $(A7)$ in the form

$$\sum_{\nu=1}^{n} \alpha_{\nu-1} y_{\nu,s} = \beta_{s-1} y_{n,m} + y_{n,s-1} \quad \ldots \ldots \quad (A7)$$

by giving s the values $1, 2, \ldots, m$. If we multiply this last equation by $\frac{(s-1)!}{(s-k_i-h)!}\lambda_i^{s-k_i-h}$ and sum between $s=k_i+h$ and $s=m$, then we obtain the following relation

$$\sum_{s=k_i+h}^{m}\sum_{\nu=1}^{n} a_{\nu-1}y_{\nu,s}\frac{(s-1)!}{(s-k_i-h)!}\lambda_i^{s-k_i-h}=$$

$$=\sum_{s=k_i+h}^{s=m}\frac{(s-1)!}{(s-k_i+h)!}\lambda_i^{s-k_i-h}(\beta_{s-1}\,y_{n,m}+y_{n,s-1}). \quad (B5)$$

Let us suppose now that $k_i+h-1<m_i$, that is $h\leqq m_i-k_i$. Now the left hand side of $(B5)$ is

$$\sum_{s=k_i+h}^{m}\sum_{\nu=1}^{n} a_{\nu-1}y_{\nu,s}\frac{(s-1)!}{(s-k_i-h)!}\lambda_i^{s-k_i-h}=$$

$$=\sum_{s=k_i+h}^{m}\sum_{\nu=1}^{n} a_{\nu-1}\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}y_{\nu,s}\lambda_i^{s-1}$$

$$=\sum_{\nu=1}^{n} a_{\nu-1}\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}O_\nu(\lambda_i)$$

$$=\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}\left[O_1(\lambda_i)\sum_{\nu=1}^{n} a_{\nu-1}\lambda_i^{\nu-1}\right]$$

$$=\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}\left[N(\lambda_i)\,O_1(\lambda_i)+\lambda_i^n\,O_1(\lambda_i)\right]$$

Also, the right hand side is

$$\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}\left[y_{n,m}\sum_{s=k_i+h}^{m}\beta_{s-1}\lambda_i^{s-1}+\sum_{s=k_i+h}^{m}y_{n,s-1}\lambda_i^{s-1}\right]$$

$$=\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}\left[y_{n,m}\sum_{s=k_i+t}^{m+1}\beta_{s-1}\lambda_i^{s-1}+\sum_{s=k_i+h}^{m+1}y_{n,s-1}\lambda_i^{s-1}\right]$$

$$=\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}\left[y_{n,m}M(\lambda_i)+\lambda_i\,O_n(\lambda_i)\right]$$

$$=\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}\left[y_{n,m}M(\lambda_i)+\lambda_i^n\,O_1(\lambda_i)\right].$$

Hence equation $(B5)$ reduces to

$$\frac{\partial^{k_i+h-1}}{\partial\lambda_i^{k_i+h-1}}\left[N(\lambda_i)\,O_1(\lambda_i)-y_{n,m}M(\lambda_i)\right]=0$$

provided that $h\leqq m_i-k_i$.

Using LEIBNIZ' Theorem and the argument of section (4), we deduce that

$$\sum_{p=0}^{h-1} \binom{k_i+h-1}{p} O_1^{(p)}(\lambda_i)\, N^{(k_i+h-p-1)}(\lambda_i) = 0$$

whence putting in succession $h = 1, 2, \ldots, (m_i - k_i)$, we have $m_i - k_i$ relations between the elements $y_{11}, \ldots, y_{1m}$ namely

$$O_1(\lambda_i) = O'_1(\lambda_i) = \ldots = O_1^{(m_i-k_i-1)}(\lambda_i) = 0. \quad . \quad . \quad . \quad (B\,6)$$

The total number of relations so obtained will be $\sum_i (m_i - k_i) = m - k$, which is the number required by the argument of section (3).

6. Our next problem is to express the elements $y_{1,1}, \ldots, y_{1,m-k}$ in terms of $y_{1,m-k+1}, y_{1,m-k+2}, \ldots, y_{1,m}$. To do this we associate with the equations $(B\,6)$ the following $k-1$ auxiliary equations.

$$\left.\begin{array}{l} -y_{1,m-k+2}\, y_{1,m-k+1} + y_{1,m-k+1}\, y_{1,m-k+2} = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots = 0 \\ -y_{1,m}\, y_{1,m-1} + y_{1,m-1}\, y_{1,m} \qquad\qquad = 0 \end{array}\right\} \quad . \quad . \quad (B\,7)$$

We now have in all $m-1$ equations connecting m unknowns. It is usual to denote the following determinants

$$\begin{vmatrix} \lambda_1^a & \lambda_2^a & \lambda_3^a & \cdots \\ \lambda_1^b & \lambda_2^b & \lambda_3^b & \\ \lambda_1^c & \lambda_2^c & \lambda_3^c & \\ & & \cdot & \\ & & & \cdot \\ \cdots & \cdots & \cdots & \lambda_t^g \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} \lambda_1^a & \dfrac{d\lambda_1^a}{d\lambda_1} & \lambda_1^a \cdots \\ \lambda_1^b & \dfrac{d\lambda_1^b}{d\lambda_1} & \lambda_2^b \cdots \\ \lambda_1^c & \dfrac{d\lambda_1^c}{d\lambda_1} & \lambda_2^c \cdots \\ & & \\ & & \lambda_t^g \end{vmatrix}$$

by $(\lambda_1^a, \lambda_2^b, \lambda_3^c, \ldots, \lambda_t^g)$ and $\left(\lambda_1^a, \dfrac{d\lambda_1^b}{d\lambda_1}, \lambda_2^c \ldots \lambda_t^g\right)$ respectively.

In the same manner we define determinants of the sort

$$\left(\lambda_1^{a_1}, \frac{d\lambda_1^{a_2}}{d\lambda_1}, \frac{1}{2!}\frac{d^2\lambda_1^{a_3}}{d\lambda_1^2}, \ldots, \lambda_2^{b_1} \frac{d\lambda_2^{b_2}}{d\lambda_2} \cdots\cdots, \frac{1}{v!}\frac{d^v \lambda_t^{g_v+1}}{d\lambda_t^v}\right)$$

which for the sake of convenience we shall write simply as

$$(a_1\, a_2\, a_3 \ldots b_1\, b_2 \ldots\ldots g_{v+1})$$

These determinants are alternants of the confluent sort.

At this point typographical difficulties make it necessary to introduce a new notation. We shall write

$$D^e \lambda_i^v \text{ in place of } \frac{1}{e!}\frac{d^e \lambda_i^v}{d\lambda_i^e},$$

From $(B6)$ and $(B7)$ we have,

$$\frac{y_{11}}{\Delta_1} = \frac{-y_{12}}{\Delta_2} = \ldots\ldots = \frac{(-)^{m-1}\, y_{1m}}{\Delta_m}$$

where, if $i \leqq m-k$ we find that $\Delta_i =$

$$\begin{vmatrix}
1 & D^1\lambda_1^0 & \cdots & D^{n_1-1}\lambda_1^0 & 1 & \cdots & D^{n_t-1}\lambda_t^0 & \cdot & \cdots\cdots \\
\lambda_1 & D^1\lambda_1^1 & \cdots & D^{n_1-1}\lambda_1^1 & \lambda_2 & \cdots & D^{n_t-1}\lambda_t^1 & \cdot & \cdots\cdots \\
\lambda_1^2 & D^1\lambda_1^2 & \cdots & D^{n_1-1}\lambda_1^2 & \lambda_2^2 & \cdots & D^{n_t-1}\lambda_t^2 & \cdot & \cdots\cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\lambda_1^{i-2} & D^1\lambda_1^{i-2} & \cdots & D^{n_1-1}\lambda_1^{i-2} & \lambda_2^{i-2} & \cdots & D^{n_t-1}\lambda_t^{i-2} & \cdot & \cdots\cdots \\
\lambda_1^{i} & D^1\lambda_1^{i} & \cdots & D^{n_1-1}\lambda_1^{i} & \lambda_2^{i} & \cdots & D^{n_t-1}\lambda_t^{i} & \cdot & \cdots\cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\lambda_1^{m-k} & D^1\lambda_1^{m-k} & \cdots & D^{n_1-1}\lambda_1^{m-k} & \lambda_2^{m-k} & \cdots & D^{n_t-1}\lambda_t^{m-k} & -y_{1,m-k+2} & \cdots\cdots \\
\lambda_1^{m-k+1} & D^1\lambda_1^{m-k+1} & \cdots & D^{n_1-1}\lambda_1^{m-k+1} & \lambda_2^{m-k+1} & \cdots & D^{n_t-1}\lambda_t^{m-k+1} & +y_{1,m-k+1} & -y_{1,m-k+3}\cdots \\
\lambda_1^{m-k+2} & D^1\lambda_1^{m-k+2} & \cdots & D^{n_1-1}\lambda_1^{m-k+2} & \lambda_2^{m-k+2} & \cdots & D^{n_t-1}\lambda_t^{m-k+2} & \cdot & +y_{1,m-k+2}\cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\lambda_1^{m-2} & D^1\lambda_1^{m-2} & \cdots & D^{n_1-1}\lambda_1^{m-2} & \lambda_2^{m-2} & \cdots & D^{n_t-1}\lambda_t^{m-2} & \cdot & \cdots\cdots -y_{1,m} \\
\lambda_1^{m-1} & D^1\lambda_1^{m-1} & \cdots & D^{n_1-1}\lambda_1^{m-1} & \lambda_2^{m-1} & \cdots & D^{n_t-1}\lambda_t^{m-1} & \cdot & \cdots\cdots +y_{1,m-1}
\end{vmatrix} \qquad (B8)$$

Expanding this determinant we find that

$$\triangle_i = y_{1,m-1}\, y_{1,m-2} \ldots y_{1,m-k+2} \sum_{\nu=m-k+1}^{m} y_{1,\nu}\,(0, 1, \ldots, i-2, i \ldots, m-k-1, \nu-1)$$

We obtain a similar expression for $\triangle_m$ but since only one element appears in the last column we get only one term on expanding: thus

$$\triangle_m = (-)^{k-1}\, y_{1,m}\, y_{1,m-1} \ldots y_{1,m-k+2}\,(0, 1, \ldots, m-k-1).$$

Hence

$$y_{1,i} = (-)^{(i-1+m-k)} \sum_{\nu=m-k+1}^{m} y_{1,\nu} \frac{(0, 1, \ldots, i-2, i, \ldots, m-k-1, r-1)}{(0, 1, \ldots\ldots\ldots\ldots, m-k-1)} \quad . \quad . \qquad (B\,9)$$

NAEGELSBACH [1]) gives a simple expression for

$$V_{i\nu} \equiv \frac{(0, 1, \ldots, i-2, i, \ldots, m-k-1, \nu-1)}{(0, 1, \ldots\ldots\ldots\ldots, m-k-1)}$$

in terms of the elementary symmetric functions $\sigma_1, \sigma_2, \ldots$ of the roots $\lambda_1, \lambda_2 \ldots \lambda_t$ of $D(\lambda) = 0$, in the case where $\lambda_1, \lambda_2, \ldots, \lambda_t$ are all different. It can easily be shown that his methods can also be applied to the cases we are considering where the λ's are not all different and that the corresponding relations hold. Thus

$$V_{i\nu} \equiv \begin{vmatrix} \sigma_1 & \sigma_2 & \ldots & \sigma_{\nu-m+k-1} & \sigma_{\nu-i} \\ \sigma_0 & \sigma_1 & \ldots & \sigma_{\nu-m+k-2} & \sigma_{\nu-i-1} \\ \cdot & \sigma_0 & \ldots & \sigma_{\nu-m+k-3} & \sigma_{\nu-i-2} \\ \cdot & \cdot & \ldots & \cdot & \cdot \\ \cdot & \cdot & \ldots & \cdot & \cdot \\ \cdot & \cdot & \ldots & \sigma_0 & \sigma_{m-k-i+1} \end{vmatrix}$$

where σ_i is the ith elementary symmetric function of the roots of $D(\lambda) = 0$ *i, e,* σ_i is the coefficient of $(-)^i\, \lambda^{m-k-i}$ in $D(\lambda) = 0$, and is therefore rational and where, since there are only $m-k$ roots of $D(\lambda) = 0$, $\sigma_{m-k+1} = \sigma_{m-k+2} = \ldots = 0$.

Hence, combining equations $(A\,11)$ and $(B\,9)$ we obtain the most general solution to the matrix equation $(A\,4)$ in terms of the σ's namely

$$Y_{n,m} = I_{n,m} \left\{ \sum_{\nu=m-k+1}^{m} y_{1,\nu} \left(B^{\nu-1} + \right.\right.$$
$$\left.\left. + \sum_{i=1}^{m-k} (-)^{(i-1+m-k)}\, B^{i-1} \frac{(0, 1, \ldots, i-2, i, \ldots, m-k-1, \nu-1)}{(0, 1, \ldots\ldots\ldots\ldots, m-k-1)} \right) \right\}.$$

[1]) Ueber eine Classe symmetrischer Funktionen Sch. Programm, Zweibrücken 1871.

7. By fitting the submatrices $Y_{n,m}$ together we can now obtain a solution to the matrix equation

$$B_S\, Y = Y B_T$$

where B_S and B_T are the rational canonical forms of the matrices S and T respectively. Suppose then that

$$W^{-1}\, S\, W = B_S \text{ and that } Z^{-1}\, T\, Z = B_T.$$

It follows that $X = W\, Y\, Z^{-1}$ is a solution to the matrix equation

$$S\, X = X\, T.$$

The solution to the problem has now been given.

Mathematics. — *Eine obere Grenze für das isoperimetrische Defizit einer ebenen Kurve.* Von O. BOTTEMA. (Communicated by Prof. W. VAN DER WOUDE.)

(Communicated at the meeting of April 29, 1933).

Ist p der Umfang, f der Flächeninhalt einer ebenen Kurve, so besteht die Ungleichung

$$\frac{p^2}{4\pi} - f \geqq 0,$$

wobei das Gleichheitszeichen nur für den Kreis gilt.

Schon vor längerer Zeit hat F. BERNSTEIN [1]) eine Verschärfung dieser isoperimetrischen Ungleichung gegeben, wobei im rechten Glied statt der Null eine Zahl steht, welche positiv ausfällt für jede Kurve, welche kein Kreis ist.

Eine weitere Verschärfung ist das Ziel einer Reihe von Arbeiten von BONNESEN [2]) gewesen. Sein Hauptresultat ist dabei die Ungleichung

$$\frac{p^2}{4\pi} - f \geqq (R-r)^2,$$

wo R den Radius des kleinsten die Kurve enthaltenden Kreises, r den Radius des grössten der Kurve einschreibbaren Kreises bedeutet.

Darüber hinaus hat BONNESEN gezeigt

$$\frac{p^2}{4\pi} - f \geqq (R'-r')^2,$$

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1) F. BERNSTEIN, Math. Ann. **60** (1905), S. 117.

2) Vgl. i. B.: Math. Ann, **84** (1921), S. 216; Math. Ann. **91** (1924), S. 252; Acta Math. **48** (1926), p. 123.

wo R' und r' die Radien des kleinsten konzentrischen Kreisringes sind, welcher die Kurve in sich schliesst. Das Gleichheitszeichen gilt dabei immer nur für den Kreis.

Daneben hat man auch gewisse *obere Grenzen* für den Ausdruck $\frac{p^2}{4\pi} - f$ (welchen wir mit BONNESEN als *isoperimetrisches Defizit* der Kurve bezeichnen) ableiten können.

So bewies KUBOTA[1]) für Eikurven die Ungleichung:

$$\frac{p^2}{4\pi} - f \leqq \frac{1}{\pi} d^2,$$

wo d den Durchmesser der Kurve bedeudet, während FAVARD[2]) die Beziehung

$$\frac{p^2}{4\pi} - f \leqq \pi R'(R' - r')$$

abgeleitet hat.

Wir beweisen den folgenden Satz:

Hat eine Eikurve mit dem Umfang p und dem Flächeninhalt f in jedem Punkt einen Krümmungsradius und sind ϱ_1 und ϱ_2 dessen obere, bzw. untere Grenze, so ist

$$\frac{p^2}{4\pi} - f \leqq \frac{\pi}{4} (\varrho_1 - \varrho_2)^2,$$

wobei das Gleichheitszeichen nur gilt für den Kreis.

Wir betrachten zwei ebene konvexe Polygone, $A_1 A_2 \ldots A_n$ und $B_1 B_2 \ldots B_n$ wo die Seiten $A_i A_{i+1}$ und $B_i B_{i+1}$ gleichsinnig parallel sind. Der durch $A_1 A_2 \ldots A_n$ festgelegte Umlaufsinn sei der positive.

Mit den zwei konvexen Bereichen A und B können wir die *lineare Schar*

$$C(\lambda) = (1 - \lambda) A + \lambda B$$

bilden. Dabei verstehen wir unter $C(\lambda)$ ein Polygon $C_1 C_2 \ldots C_n$, wo C_i ein auf der Geraden $A_i B_i$ liegender Punkt ist, sodass

$$C_i A_i : C_i B_i = (1 - \lambda) : \lambda.$$

Die Seite $C_i C_{i+1}$ liegt offenbar auf einer Geraden, welche mit $A_i A_{i+1}$ und $B_i B_{i+1}$ parellel ist.

Hat A_i die Koordinaten x_i, y_i, B_i die Koordinaten x_i', y_i', dann hat C_i die Koordinaten

$$x_i'' = (1 - \lambda) x_i + \lambda x_i' \quad , \quad y_i'' = (1 - \lambda) y_i + \lambda y_i'.$$

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[1]) KUBOTA, Science Rep. of the Tôhoku Imp. Univ. 13 (1923); The Tôhoku Math. J. Vol. 24 (1925). pg. 60.

[2]) FAVARD, Matematisk Tidsskrift B (1929), pg. 62.

Die Flächeninhalte der Polygone A und B sind bzw.:

$$F_1 = \tfrac{1}{2} \sum_1^n (x_i y_{i+1} - x_{i+1} y_i) \quad , \quad F_2 = \tfrac{1}{2} \sum_1^n (x'_i y'_{i+1} - x'_{i+1} y'_i).$$

Für den Inhalt von C erhält man

$$F(\lambda) = (1-\lambda)^2 F_1 + 2\lambda(1-\lambda) F_{12} + \lambda^2 F_2,$$

wo

$$F_{12} = \tfrac{1}{2} \sum_1^n \{ x_i (y'_{i+1} - y'_i) - y_i (x'_{i+1} - x'_i) \} = \tfrac{1}{2} \sum_1^n \{ x'_i (y_{i+1} - y_i) - y'_i (x_{i+1} - x_i) \}$$

den von BRUNN und MINKOWSKI[1]) herrührenden *gemischten Flächeninhalt* der beiden Bereiche bedeutet. Für diesen letzteren gilt bekanntlich die fundamentale Ungleichung

$$F_{12}^2 \geqq F_1 F_2,$$

wo das Gleichheitszeichen nur gilt, wenn die zwei Bereiche *homothetisch* sind. Diese Ungleichung sagt aus, dass der Inhalt $F(\lambda)$ nicht für jeden Wert von λ positiv ist. Wenn man mit der hier angewandten Methode eine *untere* Grenze für das isoperimetrische Defizit herleiten will, so hat man wie es von BLASCHKE[2]) und im wesentlichen auch von BONNESEN getan ist, verschiedene Werte von λ anzugeben, wobei $F(\lambda) \leqq 0$ ist. Wir suchen dagegen nach solchen Werten von λ, wobei $F(\lambda) \geqq$ ausfällt.

Es ist klar, dass für $0 \leqslant \lambda \leqslant 1$ der Wert $F(\lambda)$ positiv ist; $C(\lambda)$ ist dann nämlich ein konvexes Polygon dessen Seiten mit den übereinstimmenden von A und B gleichsinnig parallel sind.

Wenn die Längen der Seiten $A_i A_{i+1}$ und $B_i B_{i+1}$ mit a_i und a'_i bezeichnet werden, so findet man für die Länge von $C_i C_{i+1}$:

$$a''_i = (1-\lambda) a_i + \lambda a'_i,$$

wenn man für die Verbindungsgerade von C_i und C_{i+1} die Richtung $A_i A_{i+1}$ als die positive wählt.

Es gibt also immer solche Werte von λ, wobei $a''_i \leqq 0$ ist; er geht daraus eben die Möglichkeit hervor, dass $F(\lambda)$ negativ wird. Anderseits ist aber gewiss $F(\lambda) \geqq 0$, wenn für jeden Wert von i $a''_i \geqq 0$ ist.

Wählt man aber λ so, dass für jeden Wert von i die Zahl a''_i negativ ausfällt, dann ist das Polygon $C(\lambda)$ ein solches, dessen Seiten sämtlich mit denen von A und B ungleichsinnig parallel sind. $C(\lambda)$ ist dann aber wieder ein konvexes Polygon, wobei der Umlaufsinn $C_1 C_2 \ldots C_n$ der *positive* ist, d.h. wir haben abermals $F(\lambda) \geqq 0$.

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1) MINKOWSKI, Volumen und Oberfläche, Ges. Abh. II, S. 230.

2) BLASCHKE, Hamb. Abh. I (1922), S. 206.

Wir setzen

$$t = \frac{\lambda - 1}{\lambda}$$

also

$$F(\lambda) = \lambda^2 f(t),$$

wo

$$f(t) = F_1 t^2 - 2 F_{12} t + F_2$$

Mit Ausnahme des Wertes $\lambda = 0$, wo $F(\lambda)$ den positiven Wert F_1 hat, sind F und f gleichzeitig $> 0, = 0$ und < 0. Wenn $0 < \lambda \leqslant 1$ dann ist $t \leqslant 0$. Die Form $f(t)$ ist also positiv für negative Werte von t.

Weiter ist $f(t) \geqq 0$, wenn

$$(1 - \lambda) a_i + \lambda a'_i \geqq 0 \qquad (i = 1, 2, \ldots n)$$

also wenn

$$t \leqq 1 \text{ und gleichzeitig } t \leqq \frac{a'_i}{a_i}$$

und wenn

$$t \geqq 1 \text{ und gleichzeitig } t \geqq \frac{a'_i}{a_i}$$

$f(t)$ ist aber auch $\geqq 0$, wenn

$$(1 - \lambda) a_i + \lambda a'_i \leqq 0 \qquad (i = 1, 2, \ldots n)$$

also wenn

$$1 \geqq t \geqq \frac{a'_i}{a_i}$$

und auch wenn

$$1 \leqq t \leqq \frac{a'_i}{a_i}$$

Wir haben also

$$f(t) \geqq 0 \text{ wenn } t \leqq \frac{a'_i}{a_i} \qquad (i = 1, 2 \ldots n)$$

$$f(t) \geqq 0 \text{ wenn } t \geqq \frac{a'_i}{a_i} \qquad (i = 1, 2 \ldots n)$$

d.h.

$$f(t) \geqq 0 \text{ wenn } t \leqq \min. \frac{a'_i}{a_i}$$

$$f(t) \geqq 0 \text{ wenn } t \geqq \max. \frac{a'_i}{a_i}.$$

Die Differenz der Nullstellen von $f(t)$ ist $\frac{2}{F_1}\sqrt{F_{12}^2 - F_1 F_2}$, sodass wir die folgende Ungleichung erhalten haben:

$$F_{12}^2 - F_1 F_2 \leqq \frac{F_1^2}{4}\left[max. \frac{a'_i}{a_i} - min. \frac{a'_i}{a_i}\right]^2.$$

Dabei sind a_i und a'_i übereinstimmende Seiten der Polygone A und B. Diese Seiten verhalten sich aber wie die Radien ϱ_i und ϱ'_i der Kreise, welche bzw. die Seiten $A_{i-1}\,A_i$, $A_i\,A_{i+1}$, $A_{i+1}\,A_{i+2}$ und $B_{i-1}\,B_i$, $B_i\,B_{i+1}$, $B_{i+1}\,B_{i+2}$ an der Innenseite des Polygons berühren.

Wir haben also

$$F_{12}^2 - F_1 F_2 \leqq \frac{F_1^2}{4}\left[max. \frac{\varrho'_i}{\varrho_i} - min. \frac{\varrho'_i}{\varrho_i}\right]^2$$

wo ϱ_i und ϱ'_i die Radien übereinstimmender Berührungskreise der Polygone sind.

Wenn in den Ungleichungen das Gleichheitszeichen gilt, so ist dabei notwendig

$$f\left(min. \frac{a'_i}{a_i}\right) = 0 \qquad f\left(max. \frac{a'_i}{a_i}\right) = 0$$

d.h. also dass das Polygon $C(\lambda)$ den Flächeninhalt Null hat, wenn λ so gewählt wird, dass eine Seite null ist, und die anderen nicht negativ sind. Für diesen Wert von λ sind also die Seiten sämtlich null. Für jeden andren Wert von λ sind sie also entweder sämtlich positiv oder negativ. Die Funktion $f(t)$ hat also nur eine Nullstelle, d.h. $F_{12}^2 - F_1\, F_2 = 0$ und die Polygone sind homothetisch.

Wir können nun die abgeleitete Ungleichung auf beliebige konvexe Kurven übertragen, indem wir sie anwenden auf eine Reihe den Kurven umschriebener Polygone mit wachsender Seitenzahl. Die Werte ϱ_i und ϱ'_i nähern sich dabei den Radien der Krümmungskreise in übereinstimmenden Punkten der Kurven, d.h. in Punkten mit gleichsinnig parallellen Tangenten.

Wenden wir die Ungleichung an in dem Fall, dass die Kurve A der Einheitskreis ist, dann wird F_{12} bekanntlich gleich $\frac{1}{2}\,p$ wo p der Umfang von B ist, F_1 wird π, die Zahlen ϱ werden sämtlich 1. Schreiben wir f statt F und deuten wir max. ϱ' mit ϱ_1, min. ϱ' mit ϱ_2 an, so erhalten wir

$$\frac{p^2}{4\pi} - f \leqq \frac{\pi}{4}(\varrho_1 - \varrho_2)^2$$

wobei das Gleichheitszeichen nur gilt, wenn die Kurve ein Kreis ist.

Geology. — *The Malay double (triple) orogen.* By G. L. SMIT SIBINGA. (Communicated by Prof. G. A. F. MOLENGRAAFF).

(Communicated at the meeting of April 29, 1933).

PART III.

Tertiary structural plan of the Sunda-orogen.

The Sunda-orogen, though strongly twisted out of its original shape is much less crushed and squeezed than the Molucca-orogen. The latter had to receive the full impact of the Australian crustmovement first-hand, whereas it affected the Sunda-orogen only indirectly. On the other side the Sunda-orogen stood under the direct influence of the Asiatic continent, the crustmovement of which however in Young-tertiary time, if present, must have been very slight, in any case, as pointed out before, much less intensive than the Australian crustmovement.

The main phase of Tertiary orogenesis started feebly in the Upper-miocene on the Australian side. After the sedimentation of the Pliocene the main folding took place in the other parts of the Sunda-orogen on the Asiatic side, opposite the Sundaland. During this latest main phase of youngest Tertiary orogenesis in the Archipelago also great portions of the Neogene geosyncline situated between the Sundaland and the Sunda-orogen have been folded and brought above sealevel. The part of the geosyncline at the Southwestern border of the Sundaland is nearly entirely folded, the part at the Eastern border however is only affected by marginal folding at the side of the Sundaland and the Sunda-orogen. The latter part thus represents a less developed stage of evolution compared with the first, as its central portion (Makassar Straits) still finds itself in the geosynclinal stage of sedimentation.

The writer recently pointed out that the Young-tertiary main trend lines in the Sunda-orogen originally generated parallel or nearly parallel to the orogenetic axis. Later deviations and divergencies, as e.g. the curious plan of complementary branching trend-lines on Sumatra-Java, have been caused by subsequent bending and fracturing of the orogenetic axis. On Western Celebes the Tertiary main trends run still more or less parallel to the orogenetic axis.

As stated above the Young-tertiary orogenesis acted much less intensively in the Sunda-orogen than in the Molucca-orogen. There is no surface reaction that could supply evidence of any important Young-tertiary Asiatic continental action. On the other hand it is quite obvious that the Sunda-orogen made a considerable approach towards the Sundaland, which

is clearly evident from the folding plan of the interjacent geosynclinal belts. Apart from other considerations the movement of the Sunda-orogen is obviously demonstrated by the interference of meridional and transversal trends on Southeastern Borneo. The writer recently pointed out that this phenomenon must have been caused by two interfering directions of stress originating in the first place from the approaching Sunda-orogen and in the last instance from the Australian action.

Comparing the Tertiary tectonic plan of the Sunda-orogen with that of the Molucca-orogen it appears that there exists an intimate relation between the deformation of the first and the crush and squeeze of the latter, the Sunda-orogen showing features which are perfectly complementary to those of the Molucca-orogen. The fracturing of the Sunda Straits e.g. is complementary to the fracturing between Sumba and the submarine ridge South of Java. The curvature of the outer Banda arc induced the twist of the inner one. The squeeze of the Molucca-orogen against the Sunda-orogen resulted in the curious shape of Western Celebes. In short, all deformations of the Sunda-orogen being complementary to and induced by those of the Molucca-orogen have been caused likewise by the Australian Young-tertiary crustmovement.

Tertiary structural plan of the Pelew-orogen.

Lack of sufficient data wil make this chapter very short and confine the discussion to the Sula-Banggai-group.

On the main island Peling of the Banggai Archipelago, a striking deviation of the Miocene trends may be observed, originated by later bending of the Pelew-axis. Generated by the same forces, during the same main phase of Young-tertiary folding, the Miocene trend on Peling runs N. to S., on the opposite part of Eastern Celebes however N.E. to S.W. [22]) Peling in this way just turned about 45° since Miocene folding took place (disjunctive movement of the Pelew-orogen, complementary to the disruption of the Molucca-orogen between Buru and Tukang Besi).

The average Tertiary main trend on the Sula Is. coincides according to BROUWER [23]) generally with the orogenetic axis. The main phase of Tertiary folding, which has not been very strong, is still unknown.

Gravity anomalies.

The foregoing statements have emphasized that important horizontal displacements have taken place in the Malay Archipelago especially in

22) G. L. SMIT SIBINGA. Heeft de Banggai-Archipel in jongtertiairen tijd een afwijkende ontwikkeling gehad? Onhoudbaarheid der pliocene Molukkenbrug? Tijdschr. Kon. Ned. Aardr. Gen. Dl. 50, 1933, p. 228.

23) H. A. BROUWER. Geologische onderzoekingen op de Soela-eilanden. I. Jaarb. Mijnw. 1920. Verh. II. p. 34. (Sep.).

Tertiary time. Originally remote parts of the earth's crust have been brought into immediate proximity or into anomalous superposition; other primary adjacent parts have been disjoined. The folding overthrusting disruption, in short the disturbance of the surface layers must have been accompanied by a corresponding disturbance of the deeper layers, or rather the opposite, i.e. the disturbance of the deeper parts of the earth's crust has caused a complementary revolution in the thin surface layers. Up till now the geologist could only study the reaction in the surface layers, as far as they were accessible to direct observation. Following the deductive method he could try to get some idea about the primary processes in the deeper layers, realizing that these would be of a quite different character due to the much greater plasticity of rocks and the lack of free movement in space.

The great significance for the geologist of the maritime gravity survey of F. A. VENING MEINESZ (l.c.) in the East Indies is that he now has at his disposal a series of well observed facts regarding the primary geotectonic processes in the deeper crust in one of the most extensive fields of tectonic activity. It will be the task of both the geophysicist and the geologist to try tracing the relations that may exist between the observed positive surface reaction and the measured negative subsurface action.

The first important fact stated by VENING MEINESZ is that the Archipelago shows abnormally large isostatic anomalies and that isostasy apparently is not maintained where tectonic activity takes place, as is often supposed to be the case.

The fact that orogenesis may take place without maintenance of isostasy seems to be of paramount geological importance. It shows that the orogenetic process proceeds more rapidly than the compensation of the isostatic equilibrium disturbed by this process. Orogenesis may thus during a certain length of time call into being a state of deviation from isostasy till isostatic equilibrium is gradually reestablished. From orogenesis without maintenance of isostasy follows of necessity the discernment of the temporal permanence of isostatic anomalies. On account of this discernment and the existence of extensive disturbances in the surface layers it may not be unexpected that Tertiary tectonic activity interfered to such an extent with isostatic equilibrium that the latter at present still shows strong deviations. One might inversely draw the conclusion that the existence of recent large gravity anomalies premises extensive mass displacements in a not far remote geological past, apart from recent or subrecent tectonic activity.

A second important point stated by VENING MEINESZ is that the deviations of isostasy appear to be remarkably independent of the main topographic features. The principal strip of negative anomalies either coincides with rows of islands and submarine ridges or it runs between the islands and the continental border. The minor gravity anomalies seem

to show the same independence from surface topography though in a less degree.

The writer fully agrees with VENING MEINESZ that the negative anomalies have been caused by excessive accumulation of light surface material (sial) in the denser sima-layer. The sialic downward protuberances in the writer's opinion are the negative orogens.

The incongruity between the structure of the thin surface layers, the positive orogens, and the gravity anomalies which are generated by the negative orogens may reasonably be expected. Considering the orogenetic body as a whole, a simple computation shows that a slight asymmetry of the orogen already causes a considerable deviation of the vertical projection of the positive orogenetic axis from the vertical projection of the anomaly-axis of the negative orogen. As highly asymmetric structures have been observed in the positive orogens it may be expected that the negative orogens as well as the whole orogenetic body may show asymmetry. Moreover, the greater freedom of movement in space and the greater rapidity of the tectonic processes in the positive orogen in contradistinction with the lesser possibility of horizontal movement and greater inertness of the negative orogen imply considerable incongruities between both parts of the orogen.

A certain deviation of the anomaly-axis from the positive orogenetic axis consequently means a corresponding deviation of the negative axis and a corresponding asymmetry of the whole orogen. The greater the deviation the greater the asymmetry and consequently the horizontal displacements caused by tectonic activity in the orogen. Due to its great plasticity the asymmetry of the negative orogen may be considerable. The maximum of asymmetry will be reached however as soon as tectonic activity surpasses the strength of the orogen, in which case the texture will be disrupted. First the positive orogen will be fractured due to its greater rigidity, the negative orogen still remaining undisrupted due to its greater plasticity. This stage is obviously demonstrated e.g. by the Molucca-orogen opposite the Sahulbank. Still stronger action will finally also disrupt the negative orogen as e.g. between Buru and Tukang Besi.

A third important point is the divergent intensity of the gravity anomalies shown by the different orogens.

Considering the Sunda-orogen we have to realize that the recent gravity survey has been a "maritime" survey, so that over great distances on Sumatra, Java and Celebes no data could be obtained. The available measurements however show conclusively that the negative anomalies of the Sunda-orogen are in general much smaller than those of the Molucca-orogen, though they may reach a value which is almost of the same order as e.g. of the part of the Molucca-orogen S.W. of Sumatra.

The negative anomalies of the Molucca-orogen, in close connection with those of the Pelew-orogen, form the dominating feature of the gravity field just as the surface reactions of these orogens dominate the tectonic

plan. With regard to Eastern Celebes one has to reckon with the probability that still stronger negative anomalies may occur more landward, as its orogenetic and magmatic evolution is quite identical with other parts of the Molucca-orogen. The Molucca-orogen opposite the Australian continent most probably includes a part of the Pelew-orogen, which applies as well to the positive as to the negative part of the orogens excepted the Kei-group, where both the positive Pelew-orogen and its negative protuberance still seem to be separated from the Molucca-orogen.

The most interesting features of the gravity field are the maxima in the strip of strongest negative anomalies.

Between Batjan (Halmaheira-group) and the Bualemo-peninsula (North-eastern Celebes) the most excessive negative maxima have been measured [1]).

The field of strong negative anomalies widens here considerably and reaches from Halmaheira up to Celebes, with a transversal trend normal to the meridional trend of the main strip. The protuberance thus shows, apart from a vigorous downward deflection in the sima, a strong sideways — principally westward deflection. The plastic negative orogens seem to be pressed out aside in a westward direction prevented from bulging out eastward by the prominent part of the Australian continent. In consequence of the vigorous squeeze and bulging out sideways of the plastic negatives the more rigid positive orogens have been entirely disrupted between Celebes-Banggai at the one side and Batjan-Obi at the other side. The enormeous accumulation of lighter surface material normal to the main trend of the Molucca- and Pelew-protuberance can only have been caused by a vigorous meridional i.c. northward directed stress, which in the writer's opinion cannot have been called into being otherwise than by the intensive Australian crustmovement with Northern tendency. The lateral bulging out of the downward protuberances formes the exact negative of the westward deflection of Celebes-Banggai, the secondary eastward deflection of the Halmaheira-Archipelago and the entire disruption of the positive Molucca- and Pelew-orogens.

The complementary feature of this disruption is the squeeze of Eastern Celebes (Molucca-orogen) against Western Celebes (Sunda-orogen).

Another disruption took place between Buru and Tukang Besi, caused by the same northward directed action though proceeding in a different way. This process also is exactly reflected by the gravimetric negative. From Ceram up to Buru the Molucca-protuberance decreases gradually and disappears entirely between Buru and Tukang Besi. From Tukang Besi the gravity anomalies increase again up to Eastern Celebes.

The negative maximum East of Timor may be explained by the influence of the neighbouring prominent part of the Australian continent.

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[1]) Prof. VENING MEINESZ kindly informed the writer that the definite computation of the station East of the Bualemo-peninsula of N. E. Celebes has given a negative effect of 167 m. gal.

South of Java the negative anomalies are gradually increasing towards the West, reaching a maximum South of Western Java. S.W. of Sumatra the negative anomalies first decrease to a minimum opposite Padang to increase again up to a maximum near Sabang.

The writer recently pointed out that Sunda Straits originated through a sharp bending and subsequent fracturing of the Sunda-orogen, complicated by faulting and considerable horizontal displacement of Western Java to the South (l.c. 18). Consequently from Eastern to Western Java the Sunda-orogen approached ever more the here still submarine Molucca-orogen; i.o.w. both orogens have been more and more squeezed together, giving rise to an increasing downward protuberance which results in an increasing negative effect. This tectonic process is manifestly illustrated by the Young-tertiary tectonic plan. The forces which caused this squeeze have been induced by the same Australian action with northward tendency.

Quite the same process has taken place at the Northern part of Sumatra, where the Sunda- and Molucca-orogens are again squeezed together, due to the same action. The Molucca-orogen from Simeulu bends towards the Nicobar- and Andaman Is., while the Sunda-orogen is faulted again and moved to the West, approaching the first mentioned.

The active influence of the Australian crustmovement thus appears to be as clearly perceptible in the negative anomalies as in the surface reactions, because all excessive maxima of the Molucca- (Pelew-) protuberance have been caused by the active influence of the Australian continent. Out of the reach of Australian action no excessive anomalies have been measured as e.g. over the Philippine deep.

The intensifying and modifying of the negative Molucca- (Pelew) orogen and its gravity anomalies within the sphere of Australian action is quite in harmony with the complementary surface reactions and just what on account of the latter might be expected.

Special attention should be called finally to the fact that the Molucca-orogen with its still excessive sialic protuberance is characterized by the predominance of undersaturated igneous rocks, while the Sunda-orogen with an apparently less excessive sialic protuberance is typified by the predominance of oversaturated igneous rocks. Instinctively we feel that there must exist some intimate relation between the magmatic development of the positive and the negative part of an orogen. Though this side of the problem seems equally important and interesting as the geotectonic side it falls outside the scope of this paper.

For the time being this short discussion of some prominent features will suffice as it shows conclusively in what direction further research and further study will be necessary.

As soon as an isogam-map, worked out in detail will be available it may be expected that many other striking tectonic features of the positive orogens will show their exact gravimetric negatives.

Summary.

The East Indian Archipelago consists of a double, partly triple orogen, situated between the Asiatic and Australian continental masses. The Malay orogens are of Asiatic origin, as the Malay geosyncline, being but a part of the great Eastasiatic marginal geosyncline, originally has been a purely Asiatic one. In this first Asiatic stage of evolution, the Asiatic crustmovement still finding an unobstructed path, the Malay orogens originated in the same way as all other East-asiatic orogens. There being no conclusive reason to postulate for the Malay part of the Asiatic marginal belt a structural plan divergent to the quite regular — at present still undisturbed — East-asiatic scheme of arcuate mountain ranges, it must be assumed that the Malay orogens originally likewise have been undisturbed arcuate orogens.

Accepting the principle of continental crustmovement the disturbance of these primary arcuate orogens is to be imputed to the Australian crustmovement. With the beginning of the Australian action the second stage of evolution sets in, namely the development of the Tertiary Australo-Asiatic intercontinental geosyncline, in which stage the Malay orogens have been squeezed, crushed and highly disturbed by the Australian crustmovement. The still undisturbed parts of the primary arcuate Malay orogen show that it was a double, partly even a triple orogen.

To unravel the problem of the origin of the East Indian Archipelago it seems a question of first-order importance to reconstruct and to characterize the primary orogens.

The recent gravity survey of F. A. VENING MEINESZ disclosed the downward protuberances, i.e. the negatives of the Malay orogens. Due to their greater plasticity they are less disturbed than the positive orogens and consequently still show the original arcuate shape and the primary connections of the disrupted positives.

The negative Molucca-orogen shows larger negative anomalies than the Sunda-orogen. The excessive gravity anomalies of the first may all be explained by the active influence of the Australian crustmovement just as the excessive surface reactions. The dominant features of the gravity field appear to be the exact negatives of the dominant features of the Tertiary tectonic plan. It may be expected that also the minor tectonic features will show their exact negatives.

Geology. — *The problem of the Tectites.* By Ir. R. J. VAN LIER. (Communicated by Prof. J. VERSLUYS.)

(Communicated at the meeting of April 29, 1933).

After the first publication dated 1787 from Prof. JOSEF MAYER about the Moldavites many attempts are made to unravel the origine of the tectites. After a study of many specimens I arrived to the following suggestion, which differs from the existing theories. The shape and the sculpture of the induvidual objects are explained with a desiccation-process by the sun of a colloïdal gelmass. The progress of the desiccation relies on conformable disiccating phenomenons of other colloïdal materials than hydrogels. Laboratory experiments are not taken.

The Billitonites. I accept the gel hypothesis of N. WING EASTON published in these Proceedings of 1921 in his paper "The Billitonites". For the origin of the gelmass I assume stagnant waterpools including the weathering products of the rocks in colloïdal state in company of other "Schutzcolloïde" as humicsol or tannin, which preserve the colloïdal state till the end condition of desiccation. The climate was a wet one, but changed in a dry one. The pools evaporating will get smaller and the concentration of the colloïdal mass will rise. The pools at last will retreat to the lowest parts of the surface. We get then a number of scattered pools of different sizes over the country. At last the concentration of the syrupy liquid will become so great, that the adhesion to the soil of this viscous mass will gradually be lost in accordance with the theory of J. VERSLUYS ("De capillaire werkingen in den bodem", Amsterdam 1916). The hydrated SiO_2 particles in the gelmass are much smaller than the grainparticles of the soil. On the surface of the gelmass the capillaric forces will become negative, which cause a separation especially when air and gas can penetrate to the soilsurface. And this is the case because the gelmass includes also gasses. The soil, saturated with water including the weathering products, will liberate in the ground these gasses. (Silicification of the bedrock in Billiton is rather common). By continued desiccation the center part will loose its adhesion too. The behaviour of the desiccating mass will depend from various factors partly independant (the shape of the soil surface, the quantity and height of the gelmass a.s.o.) and partly dependant from its chemical and physical properties.

First we will get a thin film on the surface of the mass. This film will be folded and can be torn by the action of the wind. On the naked surface another film will be formed, which will become strong enough to resist the action of the wind and will separate the mass from the air. The formed

gasbubbles, caught by this film, will burst and their impression on the fluid-surface will be lost, but when the liquid is already viscous enough, their impression will be preserved. The surface of the mass will become crowded with bubbles and their impressions. When the height of the mass will become on some spots thin on account of the unevenness of the soilsurface, the surfacestresses will split the gelmass. Small masses with sphereoïdal forms with circular and elliptic horizontal sections are formed with a fluidal structure. Especially on the outline of the gelmass crumbling off will happen. The periphery of the gelmass hardens in higher degree than the rest of the gelmass and will counteract the shrinking forces. These parts can get longitudinal forms as cylinders, dumb-bells, pears and tears etc. When a part of the circumference of the gelbody has a shape as in fig. 1 and the contraction takes place in the direction of the arrow, the con-

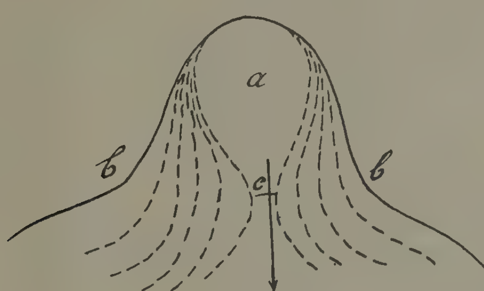

Fig. 1.

traction will be at *b* according to the dashed lines. The part *a* of the gel-mass will be cut off by *c*. By this action the cicatrices of the surface will not be only stretched in the direction of the arrow, but also the gelmass within the conical part of the body *a*. A. LACROIX writes, that in tectites of Indo China the gasbubbles in the thick part of the body were spherical and stretched in the neighbourhood and direction of the point. When *a*. is cut off, the surfaces in the neighbourhood of *c* will flow together. The viscosity of the mass will determine the shape of the broken ends. When the mass is very viscous the point of separation can be drawn to a long sharp point, which will be bent after the separation by the gravitation downward, thus in the direction of the bottom part of the tectite. This is in agreement with the observations of A. LACROIX (Comptes Rendus Tome 192, 1931 N^{0}. 26 page 1687), that the tail of the tectite is bent to the side of the surface with the smallest sculpture and which is in consequence of this theory the bottom part. The dumb-bells are formed when besides the neck at *b* a second weak point arises, where the object is cut off, (fig. 2). When the mass is liquid enough the point of section will be contracted to a round shape.

We have to examine now the liquid drops more in detail. The liquid has no adhesion to the soil. The dropbodies are held in equilibrium by the forces acting in the body (surface stresses and gravity), which give a

mathematically fixed vertical section (fig. 3). By a distinct horizontal section belongs one distinct height.

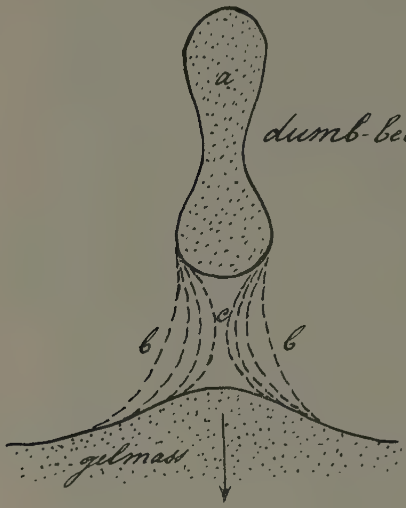

Fig. 2.

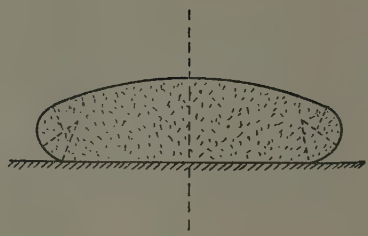

Fig. 3.

We consider now a drop of circular shape, whose top has the design of the original gelmass. The part with the smallest radius of curvature will dry the soonest, the quantity of liquid being there the smallest and the supply of heat the biggest on account of the radiation of the soil. The bottom of the drop will be covered also by a film. The whole body of the drop is then enclosed by a film. Through the desiccation the film will be subjected to contraction. The liquid mass inside the drop will get a pressure, which will be increased by the development of gas- and dampbubbles. This pressure will give the fluid body a globulous shape as far as its original shape and the gravitation will allow. The original bottom of the drop will normally be smaller than the upperpart of the body. The limit between the upper and lower part is formed by the brim, which will be later the keel. How the brim will demonstrate itself, depends from the grade of desiccation at the moment the drop gets its globulous shape. When the film of the swollen drop cracks, a part of the gas will escape and the pressure will lessen. The body will partly sink. Part of the bottom will show a flat surface, and whereas the bottom was globular and the situation of the object labil, the flattened part can be excentric.

The sculpture will depend from the speed of the desiccation. When it is going slowly by diffusion, the water and gas will escape with a small development of gasbubbles, whose impressions join the design, which the drop has got from the original gelmass. In case the film was already too strong the original design will be for the greatest part the final one and the moisture escapes by diffusion. Whereas the gasbubbles, formed inside, rise, the topside of the objects will show a stronger sculpture than the

bottomside. The bottomside is getting also a supply of heat; gasbubbles will be formed too, which will stick to the film. The bubbles, which leave the film, will rise to the top and will leave their traces behind in the viscous mass or will stop somewhere in the mass. The bubbles on the top will partly flow together or can form rows, rosettes etc. Radial arrangements with the top of the object as a center can occur. When the mass is hardened we find the prints of these gasbubbles back as fine and coarse pits, gutters, grooves etc. The thin film, which covered these cavities will become after desiccation too subtile to resist later the pressure of the covering beds.

It will not be rare, that the envelopping film bursts over a row of gasbubbles in consequence of the contraction by the desiccation. Is this crack situated on the topsurface, the edges of the crack will open. The thick fluid will stick to these edges and will stay concave. The surface of the crevasse behaves itself as the original surface. Small gasbubbles can make new prints on them. When the bursts or cracks are on the sides of the object, it is possible to get sharp cuts with rounded ends. The thick liquid will stay more or less vertical between the upper and bottom surface. By many Billitonites the edges are so thin that they become transparent (fig. 4).

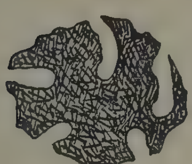

Fig. 4.

Sometimes Billitonites have deep aequatorial cuts. The upper and lower surfaces shrink. We get surface tensions with radial directions to the centres of their surfaces with the consequence that the skin can split at the aequator. While in the interior of the tectite the material is still viscous, these cracks will stop somewhere.

When a rather big gasbubble is lying under the film and the upper-part of the liquid is already hardened, the underpart of the gas-bubble will stay in the liquid mass. By the shrinkage of the kernel the lower part of the gasbubble must be pulled to the inside of the kernel and the funnel is born. It is obvious, that funnels will never intersect one another.

When several big gasbubbles settle below the film in the form of a rosette and the bubbles are pressed against one another, the upper and lower part of the bubbles are rounded. The central space between the bubbles is filled with the fluid mass with sharp edges, where two gasbubbles meet (fig. 5 and 6). When this mass hardens the problem of the mysterious tables is

Fig. 5.

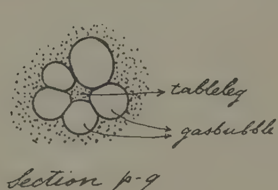

Fig. 6.

solved. When a tableleg is hardened, this will not be the case with the kernel. This kernel will shrink by loss of moisture. The leg is fixed to the

shell and will not follow the shrinkage of the kernel. Round the lower end of the leg a deepened gutter will arise.

In the case several tables are present it is clear, that the tops of the tables are lying in one bend surface, the original surface of the drop. The table tops have principally the sculpture of the original film. The Billitonites have often the tables at the keel and by elongated objects at the ends. The tablelegs, which are vertical or nearly vertical will flow in the liquid state quicklier than the inclined ones and the horizontal ones will not flow at all. The more the tableleg deviates from the vertical, the chance to harden is bigger!

In case a table leg in the fluid state cuts off and the fluid is not as liquid as to disappear totally in the kernel you get a navel.

Not only the top surface of the tectite can have many gasbubbles, also the bottomside can be occupied by them. The kernel will be connected to the shell on many or on few places. An example is given in fig. 7. This shell will be separated from the kernel by the rude treatment, to which nature self exposes the objects lateron. The kernel is found later as a tectite. The surface, which is shows, is that of the dryed kernel and may not be confounded with the original surface of the dropbody. We have Billitonites with their original surface and specimens with a surface of the dryed kernel. The separated shell is found too in the form of sherds. A. LACROIX (Comptes Rendus N⁰. 26 of 1931 pg. 1688) has got from Indo China shells sometimes as parts of a sphere and sometimes as parts of an ellipsoide.

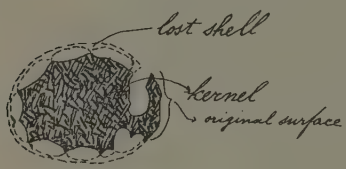

Fig. 7.

When the content of gas in the gelmaterial is high and when the shell has become too quick too tough as to allow the bubbles to pass, they can be collected to a big gasbubble inside the body of the tectite. The pressure of this gasbubble can have a great influence on the external shape of the tectite when the shell is still elastic. The greater the pressure the more the shape will be that of a sphere. I refer to the specimen of A. LACROIX (Comptes Rendus 1929 p. 285). The pressure inside was so big and perhaps also the stresses in the skin, that in the start of the attempt to cut the tectice in two pieces, the upper part was flung away.

The given genesis of the tectites makes foreign inclusions not probable.

To explain all the details of the form and sculpture of the Billitonites it was not necessary to have recourse to other forces of nature, as the action of acids.

In consequence of this desiccation hypothesis the Billitonites are formed on the spot or nearby, where they are found. They are formed in the lower parts of the country and are laying in the course of the future brooks. The objects could be transported only through the depression in which they are laying. Transport from the surrounding hills could not take place, because they could not be formed there. In Billiton people have never found a

Billitonite in the eluvial tindeposits. These eluvial exploitations are also numerous in number.

By transportation from a higher point in the valley downstream a Billitonite can be deposed on an already settled bed of sand or clay. It is not necessary to find them only on the bedrock.

The Australites. We have here to do with tectites with innate shapes. For the Australites the gelmass has to be split in drops with spheroïdal shapes in the time the adhesion with the soil existed, just the same that happens, when a solution is evaporated in a dish with a horizontal bottom too large for the small quantity of liquid. But this fact will have the greatest influence on the shape of the endproducts. In the case of the Billitonites the separated drops have a height corresponding with the horizontal diameter, in the case of the Australites the height of the drops can vary between a minimum till a maximum. The big difference between these two kinds of tectites is, that a Billitonite-drop can have only one distinct height belonging to its size of diameter, while an Australite-drop can have any amount of heights between a minimum to a distinct maximum height.

The separated geldrops will dry and also the bottom will get gradually free from the soil. But we will now consider a thick drop (fig. 8). The rim of the drop is thin and will be desiccated long before the other part of the drop and will get another appearance. It may happen also that parts of

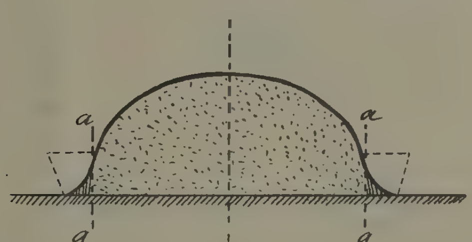

Fig. 8.

this rim and perhaps the whole rim is contracted in the body of the drop. When the thin rim breaks off later, the edges with the bottomline will be sharp. When the thin rim is not formed we will get at the ends of the bottomline curves with a small radius of curvature, which will give also a sharp borderline. In every case the angle of the flanks of the drop with the bottomline is a sharp one and will form the sharp keel of the Australites, a peculiarity of these tectites. During the hardening process the bottom part will become rounded and by specimens with great height always is the smallest part of the surface.

Now we will consider a drop with a small height (fig. 9). The rim will behave itself as the rest of the drop, because the volumes of liquid are now more comparable. The rim will generally be contracted in the drop body

(fig. 9*b*.) and will form a ringshaped body. The bending action is a stronger one for thin drops than for thick ones. It is clear, that this action will occur in successive order and is the origin of the low curved ridges on the bottom side. The toppart will be pressed together and will form concentric folds. Depending from the thickness of the drop, the grade of

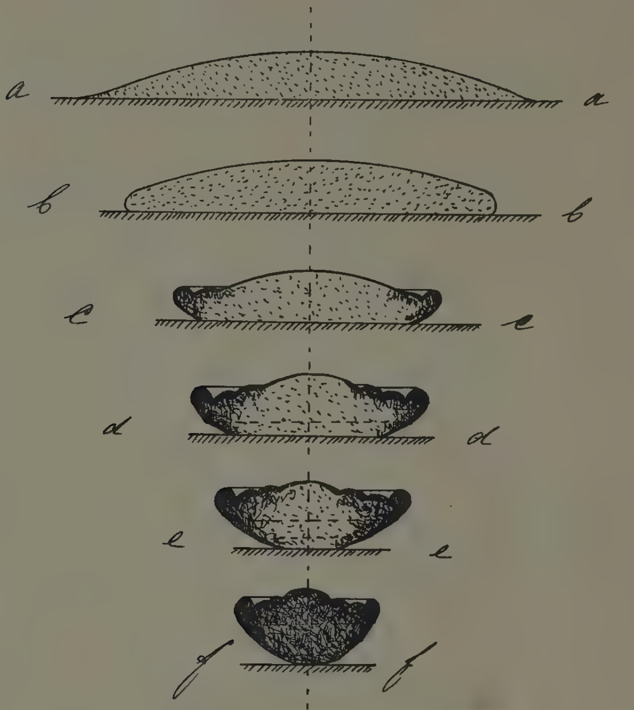

Fig. 9.

fluidity of the gelmass in the drop and the grade of convexity of the bottom, the gelmass will flow from the periphery to the center. A weakened area between the ring and the core of the tectite can arise. After desiccation this connection will be a weak one and a separation of the ring can take place. These separated rings are really found (E. J. DUNN "Pebbles" plate 58 fig. 1). Fig. 9 *a* to *f* gives the development from a drop to a buttonshaped Australite. The bottomside in this case forms the greatest part of the surface. Very often these Australites have near the top a depression, originating in the endstate of desiccation.

We can get also in the center of the core a gasbubble of big size. The higher the gaspressure inside, the more the Australite will get the form of a sphere with a keel, as the splendid Australite from Kangoro Island exhibits.

With the aid of this theory the rare specimens of Australites like a watchglass and the one DARWIN mentions can be explained without difficulty. When we have a big circular drop with a very small height, the shape of a watchglass arises by the shrinking of the top surface and the bending of

the bottom surface. When the gelmaterial inside can flow to the center, we get the Australite of DARWIN. The concentric ridges at the outside of the surface of this specimen are already explained above.

The sculpture of the Australite is very simple. Whereas they originate from drops separated in a state of high fluidity, they cannot have the original design of the gelmass in contradistinction with the Billitonites. Fluidal structure on the surface is impossible. Inside of the tectite we will have fluidal structure caused by the bending up of the bottom.

The Moldavites. The Moldavites can be divided in two groups. The biggest group is formed by the sherds. The other group, mostly originating from Mähren, have their innate shape, with other words a spheroïdal shape.

As we have seen several factors have to be taken in account for the genesis of the tectites. The gelmass can have by an advanced state of desiccation a vertical height too big to be split in small parts. The drying process takes place over the whole top surface of the gelmass. The influence of the surface stresses will disappear proportionately with the hardening of the surface. Separation of dropbodies will not easily happen but can happen. Where the mass has the smallest height, it will get till the bottom sooner a higher degree of viscosity than there, where the mass has a greater height. When the deeper parts will dry, the original horizontal top surface of the gelmass will get inclinations, dipping in the direction of the deeper parts. A gentle flow of the viscous material will take place in these directions. The surface of the gelmass was also crowded with bubbles and bubble-prints, which will be stretched by the flow in parallel grooves of different sizes. The fluidal structure is now evident. The upper surface will have a stronger sculpture than the bottom one. When the hardening process is achieved the mass will be broken to pieces through contraction, differences of temperatures a.s.o. and the places, where these fractures will occur will be the weak spots. With this sort of tectites we only have one kind of sculpture, the altered sculpture of the primitiv gelmass in contradistinction with the objects with innate spheroïdal shape. Now it is also evident, that the very fine feathershaped structure on the surface, which evokes the idea by W. HAIDINGER as to be "gleich einer abgeblasenen Milchhaut", is often met with the Moldavites and occasionly with the Australites, because they have the undisturbed original surface of the gelmass of great fluidity. That these very fine designs are preserved is a proof, that the material of the tectites is not easily attacked by the weathering agencies and that the forming of the sculpture by weathering is not probable.

As a consequence of this theory the Moldavites in sherdshape originate from larger gelpools. A great quantity is formed on one spot. It is then no wonder when they are later found abundantly on some localities as is really the case in Bohemia.

The Tectites from other localities. The tectites of most of the other localities have an innate spheroïdal shape with a sculpture comparable with the Billitonites but never as strong and varied.

The Queenstownites must be treated separately, because their shape and the localities, where they are found, differ from the other described tectites. They are found on the slopes of hills. The figures given by FR. E. SUESS ("Mitteilungen der Geologischen Gesellschaft in Wien", part VII, 1914) and his description give me the suspicion, that the Queentownites are the soldified rests of a stiffy material flowing gently downward. I suppose, that this gelmass trickled out of fissures in the rocks of the mountain and is formed in these acid rocks. While the origin of the gel is another one, the chemical composition will differ from the gelmasses from which the other kinds of tectites originate. This is indeed the case. I refer to the just mentioned paper of FR. E. SUESS. On page 110 he comes to the consequence, that the Queenstownites are an other kind of tectites.

In the course of the geological ages it will not be a rarity, that a gelmass would dry. The origin of the tectites is attributed to such a gelmass, but is it necessary, that tectites always will be born by such a desiccation? The answer of this question will be treated in the next section.

A more general application of the desiccation theory. The occurrence of tectites was formerly only known in Bohemia, Billiton and Australia. In the course of the years many other localities are discovered. And this is comprehensible, when the tectites are born out off a gelmass, originating from the weathering down of rocks. The conditions, to which a gelmass has to obey, to generate tectites, have to be still unknown but probable distinct ones. But when these conditions do not prevail, what will be the endproducts of the desiccation? These endproducts must then be found abundantly and till now not be recognised to be the desiccation product of a gelmass. As main product I keep the pebbles between the gravel in the rivers, as found in our Dutch rivers, with a spheroïdal shape and inside consisting of silicious material very oft with LIESEGANG's rhythmical precipitation running parallel with the surface. My suggestion is, that they are products of desiccation of a gelmass. The rhythmical precipitation took place when the separated dropbodies with a film came in contact with water including weatherings products. As they are formed in the lower parts of the country, they will be collected by the rivers lateron and mixed with the gravel formed in other ways. In the same way the quartz nodules with water inside, the chalcedony and agate amygdales (Uruguay) with a weak salt solution inside (synäresis?) may have been formed.

Against the developped desiccation theory some objections can be made. The first one will be, that in the geological knowledge the existence of gelpools is unknown. Silicification of rocks is a common feature caused by solutions. In case these solutions could not more penetrate into the rocks,

they can form pools and be subjected to desiccation. The geological, geographical and climatological conditions have to cooperate.

The chemical question of the tectites is still a puzzle. The general opinion is that we have to do with a glass, but is everything a glass that has the properties of a glass? Glass is a molten mixture, the proof has to been given, that the material of the tectites is a molten product and nothing else.

With this desiccation hypothesis I hope to have brought the problem of the tectites a step nearer to its solution.

Medicine. — *On the occurrence of heterophyl antigene and its importance for a specific cancer diagnosis for human beings.* By L. DE KROMME and J. R. DE BRUÏNE GROENEVELDT (from the "BRUÏNE GROENEVELDT" laboratory, Cancer Research department of the Instituut Voor Preventieve Geneeskunde, at Leiden). (Communicated by Prof. J. VAN DER HOEVE.)

(Communicated at the meeting of April 29, 1933).

In the course of the researches carried out by WATERMAN and DE KROMME in regard to the serumreaction of FREUND and KAMINER and the preparation, from the reticulo-endothelial system, of a substance capable of dissolving cancer-cells (Report Kon. Akademie van Wetenschappen, te Amsterdam, dept. Natuurk. Vol XXXVI N⁰. 3 and Biochem. Zeitschr. 205 Bd. 1—3 Heft) a new phenomenon was discovered, viz. that of the agglutination of cancer-cells.

On a drop of a cancercell-suspension, 1^{cc} being added to normal serum and being allowed to stand for some hours at 37°, it was found, that a part of these cells had been dissolved. Carcinomatous serum does not contain the substance responsible for this carcinolysis, whilst in its place a cancer cell-protecting-substance occurs, which protects the cells against the dissolving property of normal serum.

It has now been found that diluted serum can produce a very strong agglutination of the cancer cells. This phenomenon could be systematically and regularly produced and made accessible for accurate analysis.

The technical side of this question was fully dealt with in the "Zeitschr. für Krebsforschung" (29 Bd. 4 Heft). Briefly stated the following description can be given: to 1 c.c. of the serum-dilution (diluted with isotonic phosphatebuffer-solution after Sörensen; p_H 7.7), a drop of a freshly prepared cancercell-suspension (washed in FREUND solution; p_H 5.4) is added. After shaking, the testtube is allowed to stand for some time (8—24 hours) at 37°. The agglutination manifests itself by a strong irreversible clotting together of the cancer-cells, giving a marcroscopic and clearly perceptible binding and adsorption of the serum colloids.

The experimental conditions are particularly sterility, exact p_H, immediate use of the fresh suspension, etc.

By this simple test we were unable to ascertain any difference of agglutinating property between normal- and Ca.-serum. In the later attempts, made at the "BRUÏNE GROENEVELDT" laboratory, to arrive at a differentiation of the sera by changing the conditions, it could only be ascertained that normal serum does not behave in any other way to Ca.-serum, neither in titre-proportion, nor in the speed of the agglutination process, nor in any other respect, under all (but for the rest mutually equal) conditions.

Under certain conditions cancer-cells (of spontaneous and also of experimental tumors of persons and animals) were always found, in respect to human blood serum, to be agglutinable.

Of especial interest, however, was the fact that the agglutinating property of animal sera is very divergent.

WATERMAN and DE KROMME found the agglutinating property of some species of the animal kingdom to be always positive, whilst certain animals appeared to vary in this respect. The grouping of this property in the animal kingdom thus appeared to be independent of the species.

With the human being, whose serum always showed the strongest action, we can also include the rabbit, the rat, the cow, the pig (and according to later researches, the monkey and the goat). The cavia, to which we can add the horse, the cat, the mouse (and according to later researches, the chicken and dog) were negative.

The researches of WATERMAN in this direction had reached this stage when the "BRUÏNE GROENEVELDT" laboratory also took up this problem. JULIUS thereby directed i.a. his attention to the agglutinability of other cell types (Ned. Tijdschr. voor Hyg., microbiologie en serologie).

Using suspensions of liver, kidney and lung cells of various animals, he found that this special grouping of the agglutinating property for the various animal species was by no means specific for the cancer-cell.

Normal body cells of cavia, mouse, chicken etc. also appeared to be agglutinable and particularly for the same animal sera, which also agglutinate cancer cells. On the other hand the body cells of those animals, which agglutinate cancer-cells, could not themselves be agglutinated by the above mentioned sera.

JULIUS also found a connection between the classification of the animal groups according to this agglutinating property and the immune biological classification of the animals according to the distribution of the heterogeneous anti-body and heterophyl antigene, as given by FORSSMAN and others.

Subsequently he noticed that the animals which agglutinate cancer-cells are in fact those which belong to the group possessing heterogeneous anti-bodies and that in addition to cancer-cells, also the body cells of those animals, whose body cells contain the heterophyl antigene, are agglutinated. Rabbit, rat, cow, pig, belong to the rabbit group (heterog. anti-bodies) and cavia, horse, cat, mouse, belong to the cavia group (named after their prin-

cipal representatives). The supposition that the agglutinating property of the normal human serum (rabbit group) is directed against the heterophyl antigene, thus appeared to JULIUS to be justified. This is in agreement with the researches of LEHMANN—FACIUS (Zeitschr. für Immun. Forsch. Bd. 56) according to which the carcinoma of the human being, as also the foetal tissues, contain heterophyl antigene.

After an extensive series of experiments (the details of which are to be reported in the American Journal of Cancer) we were able to confirm that the occurrence of heterogeneous anti-bodies and the agglutinating property on the one hand, and the containing of heterophyl antigene and the agglutinability of the same body cells (as also all cancer-cells) on the other hand, are in fact inherent properties.

By flaking out of alcoholic tumor cel-extractions, after the addition of rabbit-antiserum (by immunization obtained with sheeps blood corpuscles), we also succeeded in identifying the particular cell receptors which, in the agglutination test, react with the respective anti-bodies as heterophyl antigene.

In the light of these new facts we are now quite able to understand why it was possible to increase the agglutination-titre on the rabbit by the introperitoneal injection of a cancercell-suspension (as was known from our previous experiments), whereas all attempts at immunization with the cavia failed. It is also clear to us that although very young individuals possess a high agglutination titre, we could never succeed, by means, of the original agglutination process, in showing agglutinants in navel string blood. The explanation of this is apparently to be found in the high content of heterophyl antigene of the foetal tissues.

In respect to this very marked difference in the two animal groups in so far as concerns the system of heterophyl antigene and heterogeneous anti-bodies, we endeavoured first to find a further explanation and assumed the hypothesis that ab. origin, both components were present in all animals, but that from special causes, at present unknown to us, an equilibrium occurred, by which, with some animals the antigene component and with others, the anti-body component, could, result as the quantitatively dominating factor.

It was à priori conceivable that e.g. in the body of cavia, agglutinants could be formed, but which, in the body cells rich in heterophyl antigene, would be immediately bound, or that both components could be present side by side in the blood, without being able to show the specific binding in reaction. Inversely with the rabbit, normal heterophyl antigene could be formed.

In our attempts to obtain concentration and isolation of the agglutinant we were able to identify this as euglobulin. The easy solubility of the heterophyl antigene in alcohol enabled us in a simple manner, to break up any antigene — anti-body — binding whic might occur. In this way, however, we did not succeed in showing either heterophyl antigene in cells of the human being and rabbit, or heterogeneous

anti-bodies in the blood of the cavia, although the specific binding occurring by the agglutination in vitro of a cancer-cell, resp. cavia-kidney-cell suspension can, by means of alcohol, easily be split up again into its two components.

With these considerations we do not wish, for the time being to venture upon the deeper biological significance of heterogenetic anti-bodies, or that of the cancer aetiology.

Our researches have shown us that the distribution of the heterogenitic anti-body in the animal kingdom occurs according to a fixed plan and that this special plan of distribution is strictly adhered to by nature. It is for us a fortunate circumstance that the human being belongs to that group which does not normally contain any heterophyl antigene. Consequently the human cancer-cell, in its property of specific anti-body binding, is firstly immune-biologically differentiated from the normal human body cell and secondly the condition of cancer in a human being is characterized in the same sense by the fact that besides normal anti-body, also heterophyl antigene is present in the same organism, which, however, does not occur with the normal being.

It is a matter of course that this special proportion in the human body can be taken as specific for cancer only when it is apparent that the property of this antigene production is not obtainable in any other pathological case.

Our provisional examination strongly points in the direction that, at least for lues and tuberculosis, no antigene is to be found. If experiments should extend and confirm these facts, we consider, in view of the marked antigene proportions, that the possibility of a specific cancer reaction of the human being will be opened.

The lines upon which this examination could be carried out are various and can, in the first place, be based on the fact that the specific cancer cell receptors are extremely loosely attached to the cell and easily enter the blood circulation.

By the use of the thermostability and the alcohol solubility of the heterophyl antigene, attempts were made in our laboratory to isolate this from the blood and the urine of cancer patients and, to react by means of the known specific, binding.

Experiments are also being carried out to obtain antiserum from a rabbit by the introperitoneal injection of urine of cancer patients with non-antigene-containing serum globulin.

SUMMARY.

1. Blood serum of the human being contains heterogenetic anti-bodies. The normal human body cells do not contain any heterophyl antigene.

2. Cancer-cells contain heterophyl antigene, in consequence of which heterogenetic anti-bodies and heterophyl antigene are simultaneously

present in the bodies of cancer patients. For human beings this condition appears to be specific for cancer.

3. The occurrence of heterophyl antigene in the blood of cancer patients opens the possibility of a specific cancer diagnostic (for human beings).

4. The realization of this possibility is closely connected with the technical elaboration of the proof of heterophyl antigene in the blood, and/or, urine. Experiments in this direction are now being carried out.

CORRIGENDUM

Proc. Academy Amsterdam Vol. 35, p. 610, Tableau II au lieu de „at" lire „atm. norm".